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FLEXIBLE MANUFACTURING SYSTEMS OPTIMIZATION APPROACH BASED ON LABOR COST MINIMIZATION

Cadet David JOSEPH

LOSI / Université Technilogique de Troyes
12, Rue Marie CURIE
10010 Troyes Cedex 01 - France
cadet.joseph@utt.fr

Alexandre SAVA, Nathalie SAUER

LGIPM / Enim - Université Paul Verlaine
Île du Saulcy
57045 Metz Cedex 01 - France
sauer@univ-metz.fr

ABSTRACT: *The research work presented in this paper deals with resources allocation in a production system in order to meet a given production specification. More precisely, we focus on human resources optimisation in design phase of a manufacturing system. We consider that operators may have different qualifications. A highly qualified operator is more expensive, but it is more efficient too. Firstly, we propose a linear programming based approach to calculate the appropriate number of operators and the requested level of skills in order to minimize labor costs for a given productivity. The result is used to generate a scheduling which minimizes the work in process.*

KEYWORDS: *Resource allocation, labor cost, linear programming, scheduling.*

1 INTRODUCTION

Managing the resources to meet given production objectives with reasonable costs is one of the main goals while designing production systems. Several researches were conducted in order to bring methods able to bring practical solutions to these problems. Interesting approaches were proposed in (Hillion and Proth, 1989), (Valentin, 1994), (Proth and al., 1997), (Korbaa and al., 2002), (Sauer, 2003), (Hsu and al., 2007).
In (Hillion and Proth, 1989) the authors proposed an approach to solve an optimization problem which consists in minimizing the work in process (WIP) maximizing throughput in a flexible manufacturing system FMS. This approach is based on analyzing the elementary cycles in the event graph modeling the FMS. By proving scheduling feasibility at minimal cycle time, they defined an exact method for event graph marking optimization and a scheduling heuristic to reach a given productivity while minimizing WIP.

The approach proposed in (Proth, Sauer and Xie, 1997) deals with minimizing the number of AGV (Automated Guided Vehicle) in the design phase of a FMS. The authors solve a strongly connected event graph marking optimization problem using Branch and Bound algorithm. Thus, they reduce the complexity regarding the method defined in (Hillion and Proth, 1988). The greedy algorithm they propose allows controlling the gap between computed solutions and optimum in order to get fast approximated results.

(Valentin, 1994) improves the scheduling approach given in (Hillion and Proth, 1988) by associating availability intervals to each machine when scheduling the operations. This approach focuses on productivity optimization while minimizing WIP is not a priority. Besides, the cyclic aspect of the problem is only partially considered since scheduled operations are not allowed to overlap the cycle time.

(Korbaa and al., 2002) defined a cyclic scheduling algorithm to minimize WIP within the cycle time. Cycle time overlapping is allowed. As a result, they obtained good solutions that, generally, tend to the optimum and always generate an admissible schedule. Although the computed heuristic is inefficient for complex problems, it allows significant time savings compared to the methods provided in (Hillion and Proth, 1988).

(Hsu and al., 2007) deals with cyclic scheduling optimization problem for FMS. Allowing some flexibility in operating sequences and in the use of transportation resources, this publication proposes an algorithm to minimize WIP while maximizing throughput based on genetic algorithms.

(Sauer, 2003) proposes an algorithm intended for marking optimization of weighted marked graphs. This modeling tool is convenient to represent systems with batches, bulk services etc. Based weighted marked graphs properties, the author develops a fast heuristic that provides quite near optimal solutions. Moreover, she establishes a necessary and sufficient condition to obtain a feasible solution for any marking optimization problem concerning weighted marked graphs with deterministic times.

Recent research work takes into account human resources and their skills in scheduling problems (Bellenguez-Morineau and Neron, 2007), (Bellenguez-Morineau and Neron, 2008), (Drezet and Billaut, 2008a), (Drezet and Billaut, 2008b). These approaches deal with project scheduling problems, which are typically not cyclic.

To our knowledge, researches conducted on FMS optimization and based on events graph modeling, so far, usually, aimed to minimize transportation resources (i.e. minimize WIP) cost while maintaining a desired level of productivity in more or less different contexts. The labor cost minimization criteria was not taken into account.

In this paper, we enhance the optimization problem presented in (Hillion and Proth, 1988) by considering human resources allocation. We suppose that each machine needs the intervention of an operator. Therefore, one might have to take labor costs in designing the production systems. Operators may have different skills defined by the ability to operate several machines. However, it is obvious that a highly qualified operator is more expensive. The aim of this research work is to develop an exact method to configure the manpower needed to achieve given a production objective for the production system presented in (Hillion and Proth, 1988). The approach that we propose is based on linear programming technique. This result is then used to compute a feasible schedule which minimizes the WIP.

The paper is structured as follows: context and assumptions are presented in section 2. The problem formulation is given in section 3, and some applications are presented in section 4. Concluding remarks are given in section 6.

2 CONTEXT AND ASSUMPTIONS

Our study is concerned with flexible manufacturing systems (FMS) which consist of cyclic production systems and where several types of products are processed. The products are brought to each machine by transportation resources (pallets, AGV…). Those FMS are represented by flow shops, job shops and assembly systems. We consider that tasks are not preemptive and each machine performs operations in conformity with given operating sequences. As stated in the previous section, the research work presented in this paper adds a new dimension to the problem considered in (Hillion and Proth, 1988) by considering the optimization of labor cost.

The analytical approach that we build to solve this problem is derived from the event graph model of the FMS. Therefore, we recall the definition of this model and the properties we relied on.

A Petri net is a directed graph consisted of two types of nodes called places and transitions. Let $\mathbf{N}= (P, T, F, a)$ be a Petri net, where $P = \{p_1, p_2, …, p_n\}$ is the set of places; $T = \{t_1, t_2, …, t_m\}$ is the set of transitions and $T \cap P = \emptyset$; F is the set of directed arcs which connect the places to transitions and vice versa such that $F \subseteq (P\times T) \cup (T\times P)$ and a is the function of arcs weights. Further, we will note $t\bullet$ (resp. $\bullet t$) the set of output (resp. input) places of transition t and $p\bullet$ (resp. $\bullet p$) the set of output (resp. input) transitions of place p.

A Petri net marking is represented by a function M: $P \rightarrow IN$ that associates a non negative integer number of tokens to each place.

An event graph is particular class of Petri net where each place has exactly one input transition, one output transition and all arcs are weighted to one. An event graph is strongly connected when there is a directed path that connects each node to another one.
The following event graphs properties are important to our work:

1) The number of tokens in an elementary circuit is invariant during its evolution.
2) A strongly connected event graph is live if and only if each one of its elementary circuit has at least one token.

We consider that the temporizations associated to tasks durations are modeled by deterministic temporizations associated to transitions and the event graph modeling the FMS is live and strongly connected. Transitions are fired as soon as possible. Under these assumptions a cycle time can be calculated for each elementary circuit γ at a steady state. It is computed based on this formula:

$$C(\gamma) = \frac{\mu(\gamma)}{M(\gamma)} \forall \gamma \in \Gamma$$

where Γ is the set of elementary circuits in the event graph; $\mu(t)$ is the firing time associated to transition t; $\mu(\gamma)$ is the sum of all firing times such that $\mu(\gamma) = \sum_{t\in\gamma} \mu(t)$ and $M(\gamma)$ is the number of tokens of an elementary circuit γ, i.e. $M(\gamma) = \sum_{p\in\gamma} M(p)$.

A circuit is critical if it has the biggest cycle time: $C(\gamma^*) = \text{Max}_{\gamma\in\Gamma} C(\gamma)$

where γ^* is a critical circuit and $C(\gamma^*)=C^*$ is the event graph cycle time.

In the literature, deterministic timed event graph are generally used to solve optimizing problem under the following assumptions:

- The market demand is static and it is described by production ratios.
- The number of transportation resources is large enough to reach desired productivity.
- The transfer times from one machine to another are insignificant.
- A given transportation resource is dedicated to one type of products and one process circuit and can carry only one product at a time.
- A free transportation resource is reused as soon as possible.
- Operations times are deterministic and there is no setup time.
- Operations are performed as soon as possible.
- Operating sequences, machines characteristics and their number are known.

3 PROBLEM FORMULATION

The optimization problem that we address deals with FMS characterized by resources with different skills and costs. Most of the approaches dedicated to resource allocation in FMS focus on optimizing transportation resources because they are usually more expensive compared to other resources. However, in western countries, the cost generated by human resources is rather important. Therefore, our aim is to compute the right number of operators with the appropriate qualifications in order to reduce the production cost. Then, the information about the available human resources is used to schedule tasks in order to minimize transport resources.

Indeed the manpower may vary between two extreme configurations. In the first one, the production system may function with one highly qualified operator which is able to operate all machines. The second one is characterized by a number of operators equal to the number of machines, each operator being able to use one machine. The right configuration is to be chosen taking into account the cost structure and the productivity requirements.

The level of skills (i.e. qualification) is defined by the number of machines an operator is able to use. Thus, an operator with a level of skills (qualification) equal to 3 is able to use 3 machines. If the level of skills is 0, then the operator is practically not used in the production system.

Based on that specific goal, we set the limits of our study by introducing additional assumptions besides usual ones. In fact, we assume that:

- there are at most as many operators as machines;
- a referred operator in our model is not necessarily assigned in the production system. An operator with a level of skills equal to 0 is called *virtual*. Virtual operators may be referred to for the needs of the mathematical model.
- an active operator (not a virtual one) is qualified for at least one machine;
- each process consists of at least two operations and operating sequences are not defined.

Taking any job shop system into consideration, we propose a method in two steps to solve the optimization problem. Firstly, a linear programming optimization approach is used to compute a lower bound for the number of operators. This information is used in the second step to minimize the work in process (WIP).

3.1 Model for labor cost minimization

The following model gives a lower bound of the number of operators needed to obtain a cycle time lower than a given value.
We build the linear program to solve our optimization problem based on the event model of a FMS. Details on modeling FMS using event graphs can be found in (Hillion and Proth, 1988).

The characteristics of the mathematical model we propose are given in the sequel:

***Notations*:**

- q: number of machines.
- C: desired cycle time such that $C \geq C^*$. C^* is the minimal cycle time that is the shortest time it takes to manufacture a product in the system. It is defined by the operating time of the critical(s) machine(s).
- p: number of processes which are illustrated by process circuits of event graphs (see figure 2). We consider that there are as many manufacturing processes as process circuits in the event graph.
- O: biggest number of operations a process may have. For all the processes, we set O as the default number of operations. Also, we assume that all the operations are distinct even when processes are duplicated for production ratios requirements (see an example of process duplication in figure 2).
- $\mu(o_{ij})$: duration of operation j belonging to process i. An operation is represented by a firing a transition (for illustration, see figure 2)
- $m_{ijn} = \begin{cases} 1, \text{ task } j \text{ of process } i \text{ is allocated to machine } n \\ 0, \text{ otherwise} \end{cases}$
- s_l: cost of an operator which was a level of skills equal to l (i.e. can be assigned to l machines). We set $l = 1,..,q$; $l = 0$ means that the operator is not actually used in the system and does not generate any cost.

Variables:

- $w_{kl} = \begin{cases} 1, \text{ operator } k \text{ must have a level of skills equal to } l \\ 0, \text{ otherwise} \end{cases}$

  $x_{ijk} = \begin{cases} 1, \text{ operator } k \text{ is assigned to task } j \text{ of process } i \\ 0, \text{ otherwise} \end{cases}$
- $y_{kn} = \begin{cases} 1, \text{ operator } k \text{ is assigned to machine } n \\ 0, \text{ otherwise} \end{cases}$
- $v_{kl} = \begin{cases} 1, \text{ skills level of operator } k \text{ is not greater than } l \\ 0, \text{ otherwise} \end{cases}$
- $z_{kl} = \begin{cases} 1, \text{ skills level of operator } k \text{ is not less than } l \\ 0, \text{ otherwise} \end{cases}$

  where: i=1,…,p (p =number of processes); j=1,…,q; k=1,…,q; n=1,…,q; l=1,…,q.

Objective function

$$Min\ z = \sum_{k=1}^{q} \sum_{l=1}^{q} s_l \times w_{kl} \quad (1)$$

We point out that, according to their definition, variables v_{kl} and z_{kl} can not be equal to 0 simultaneously.

We assume that any operator's cost is determined by his level of skills. The cost of an operator k whose competence level is $l \in \{1,\ldots q\}$ is given by $\sum_{l=1}^{q} s_l \times w_{kl}$. In order to get the cost of all the operators assigned or not, we use the expression (1). The labor cost is minimized under the following constraints.

Constraints

- $\sum_{k=1}^{q} x_{ijk} = 1$ (2)
- $y_{kn} \geq x_{ijk} m_{ijn}$ (3)
- $\sum_{i=1}^{p} \sum_{j=1}^{O} x_{ijk} \mu(o_{ij}) \leq C$ (4)
- $z_{kl} - \left(\frac{\sum_{n=1}^{q} y_{kn} - l}{q} \right) \leq 1$ (5)
- $\left(\frac{\sum_{n=1}^{q} y_{kn} - l}{q} \right) - z_{kl} < 0$ (6)
- $v_{kl} + \left(\frac{\sum_{n=1}^{q} y_{kn} - l}{q} \right) \leq 1$ (7)
- $\left(\frac{l - \sum_{n=1}^{q} y_{kn}}{q+1} \right) - v_{kl} < 0$ (8)
- $w_{kl} = v_{kl} + z_{kl} - 1$ (9)

where: i=1,…,p; j=1,…,O; k=1,…,q; l=1,…,q; n=1,…,q.

Constraint (2) shows that a task is done by only one operator. It ensures that all tasks are assigned and all operators have disjunctive tasks. In expression (3), an operator k is assigned to a machine n if he performs at least one task using this machine. In (4), we make sure that cycle time is not violated by forcing all tasks assigned to any operator to be done within cycle time. Constraints (5), (6), (7) and (8) oblige variables v_{kl} and z_{kl} to take values according to their definition when we compare the actual skills level of an operator k to all possible values of l. As shown in equation (5), z_{kl} = 0, if the number of machines which operator k is assigned (i.e. $\sum_{n=1}^{q} y_{kn}$) is less than the value of l into consideration, otherwise z_{kl} = 1 as shown in(6). On the other hand, in equation (7), v_{kl} = 0, if the number of machines which operator k is assigned is greater than the value of l into consideration, otherwise v_{kl} = 1 as shown in (8). The equation (9) applies the definition of w_{kl} to its mathematical expression. From the equation, it results that:

- w_{kl}=1 if operator k is required to have a level of skils equal to l;
- w_{kl}=0, otherwise.

Property: *The solution provided by this model is admissible.*

Proof: First of all, the model is based on event graph mathematical properties. For a given job shop, the solution we search for is intended for determining control circuits which allow the human resources utilization with the lowest cost possible. By requiring mutual exclusion of operators on the same task, obtain control circuits which can be fully illustrated by event graph formalism.

In fact, in (Hillion and Proth, 1989), it is proved that it is always possible to achieve the minimum cycle time under a functioning where transitions fire as soon as they are enabled. Let us assume that there are enough transportation resources. The necessary and sufficient condition for reaching minimal cycle time is that, at least, one circuit is critical after scheduling. Thus, in any event graph resulting from our mathematical model, it is possible to saturate at least one control circuit. Furthermore, by not allowing any control circuit cycle time to be greater than desired production cycle time through constraint (4), we ensure that, in the most extreme case, even for the minimum cycle time, we can barely saturate all the control circuits. Indeed, we infer that any solution from our model is admissible since, with enough transportation resources, even in case of minimal cycle time, scheduling is feasible.

3.2 Model for scheduling and WIP minimization

The human resources configuration obtained at the previous step is used to compute an optimal task scheduling which minimizes the work in process. The method we use to solve this point is based on the approach given in (Bourdeaud'Huy and al., 2006). This approach deals with cyclic scheduling problems which aim to minimize WIP. The WIP is calculated based on two binary variables. The first one, α_{ij}, is set to 1 if the task succeeding task j of process i starts before the end of it. In other words, for each pair of consecutive operations in a process that is considered, this variable reflects whether or not the precedence constraint is violated. An example of calculating α_{ij} is given in table 1, based on the scheduling illustrate in figure 1. It is admitted that, even if all operations of a process are sequenced in the order defined by the process routing, there is a violation of the precedence constraint between the last and the first tasks. Thus, each routing needs at least one transport resource (i.e. one unit of WIP). By associating starting times of all operations to the same cyclic time interval [0, C], we observe that the first task which is considered to be preceded by the last one starts before the ending time of its predecessor. Based on the principle of precedence constraint violation, the model will generate a sufficient number of WIP to ensure that the processes execution satisfy the given cycle time.

Furthermore, β_{ij} is used to take into account WIP due to precedence constraint violation when an operation overlaps the cycle time. A cycle time overlapping occurs when the starting time of a task added to its duration returns a higher value than cycle time that means. In this case, the task is completed in the next cycle. Then, if the end time of the overlapping task is preceded by starting time of the successive one, a precedence constraint is violated. Solving this problem consists in increasing the WIP with one unit. (see table 1 for illustration).

Process example	Two consecutive tasks	α_{ij}	β_{ij}	WIP
Figure, cycle time C = 8	$o_{11} \rightarrow o_{12}$			1
	$o_{12} \rightarrow o_{13}$	×		
	$o_{13} \rightarrow o_{11}$			
Figure	$o_{11} \rightarrow o_{12}$	×		2
	$o_{12} \rightarrow o_{13}$			
	$o_{13} \rightarrow o_{14}$	×		
	$o_{14} \rightarrow o_{11}$			
Figure	$o_{11} \rightarrow o_{12}$	×	×	2
	$o_{12} \rightarrow o_{13}$			
	$o_{13} \rightarrow o_{11}$			

Table 1: WIP size determination for some scheduling examples

The notation $(j\%O)+1$ represents the operation which succeeds o_{ij} in the process i. This notation was proposed in (Bourdeaud'Huy and al., 2006) and the symbol % is the operator "modulo".

<u>Data</u>

- x_{ijk} : human resource allocation calculated previously
- $B \geq 2C-1$: a constant used to make sure that some inequality relations are verified (Bourdeaud'Huy and al., 2006)

<u>Variables</u>

$t_{i,j}$: starting time of operation o_{ij}

$$\alpha_{ij} = \begin{cases} 1, \text{if } t_{i,j} + \mu(o_{ij}) \geq t_{i(j\%O)+1} \\ 0, \text{otherwise} \end{cases}$$

$$\beta_{ij} = \begin{cases} 1, t_{i,j} + \mu(o_{i,j}) \geq C \wedge (t_{i,j} + \mu(o_{i,j}))\%C \geq t_{i,(j\%O)+1} \\ 0, \textit{otherwise} \end{cases}$$

$$\theta_{ij}^{i'j'} = \begin{cases} 1, \text{if } t_{ij} < t_{i'j'} \\ 0, \text{otherwise} \end{cases}$$

<u>Objective function</u>

$$Min\ Z' = \sum_{i=1}^{p} \sum_{j=1}^{O} (\alpha_{ij} + \beta_{ij}) \quad (10)$$

<u>Constraints:</u>

$$C - 1 \geq t_{ij} \geq 0 : \quad (11)$$

$$t_{ij} + \mu(o_{ij}) \geq 1 + t_{i,(j\%O)+1} + B(\alpha_{ij} - 1) \quad (12)$$

$$t_{ij} - t_{i,(j\%O)+1} - B\alpha_{ij} \leq -\mu(o_{ij}) \quad (13)$$

$$t_{ij} - t_{i,(j\%O)+1} - B\beta_{ij} \geq C + 1 - B - \mu(o_{ij}) \quad (14)$$

$$t_{ij} - t_{i,(j\%O)+1} - B\beta_{ij} \leq C - \mu(o_{ij}) \quad (15)$$

$$(x_{ijk} x_{i'j'k} + m_{ijn} m_{i'j'n})(t_{ij} - t_{i'j'} + \theta_{ij}^{i'j'} C) \leq (-\mu(o_{ij}) + C)(x_{ijk} x_{i'j'k} + m_{ijn} m_{i'j'n}) \quad (16)$$

$$(x_{ijk} x_{i'j'k} + m_{ijn} m_{i'j'n})(t_{i'j'} - t_{ij} - \theta_{ij}^{i'j'} C + \mu(o_{i'j'})) \leq 0 \quad (17)$$

where: $i,i'=1,\ldots,p$; $j,j'=1,\ldots,O$; (i,j)<(i',j')

Constraint (11) shows that all tasks start within the cycle time interval. According to constraint (12), α_{ij} is set to 0 when precedence constraint between operations O_{ij} and its successor is respected. Otherwise, α_{ij} is set to 1 by constraint (13). In constraint (14), β_{ij} is set to 0 when task O_{ij} overlaps cycle and precedence order between tasks O_{ij} and $O_{i(j\%O)+1}$ is respected. But, when there are cycle overlapping and precedence order violation, β_{ij} is set to 1 through constraint (15). Constraints (16) and (17) are effective only when two operations o_{ij} and $o_{i'j'}$ are affected to the same operator and/or to the same machine.

4 APPLICATIONS TO JOB SHOPS

For applications purposes, we adopt a cost structure which defines the cost associated to each operator according to his qualification. Specifically, for skills levels ranging from 1 to q, the associated salary is defined by multiplying given coefficients {f_1,…, f_q} with a base salary, denoted ct_1. We assume that the salary of an operator with a level of skills equal to 1 is a basic salary. Then f_1 is equal to 1. Let's emphasize that in our context, salary stands for the total cost of using an operator by a company. For followings applications we consider five levels of skills: f_2 = 1.2, f_3 = 1.4, f_4 = 1.55 and f_5 = 1.7. To illustrate the results of our method, we applied it through several job shops examples.

4.1 Example of job shop inspired from Hillion and Proth (1988)

Let's consider a job shop (job shop 1) running at a desired cycle time of 8 minutes (C = 8) with three machines and three types of products. The different routings are:

- $Prod_1$: M_1(1), M_2(3), M_3(3) ;
- $Prod_2$: M_3(1), M_2(2) ;

- $Prod_3$: M_1(2), M_3(1).

Values between parentheses correspond to machines processing times (in minutes) for each type of products.

The production ratios are thus given: 25% products type $Prod_1$, 25% product type $Prod_2$ and 50% product type $Prod_3$. An event graph model of job shop is shown below (figure 2). Through the figure, only the process circuits must be considered because, control circuits which represent tasks execution order on the machines are not known. In fact, the control circuits correspond to a given schedule. Operations are represented by transitions, the WIP by places p_i and the transportation resources by places pr_i.

Each process circuit corresponds to the routing associated to a given type of product. Process circuits are duplicated in order to respect production ratios. During one cycle must be produced twice more products type $Prod_3$ than $Prod_1$ and $Prod_2$. Thus, the model of the production system contains two production circuits for products type $Prod_3$, one for products type $Prod_1$ and one for product types $Prod_2$.

We implement our optimization method using Xpress-MP solver to solve this problem and we obtain the following results in terms of human and transportation resources optimization, assignment and scheduling.

Human resources optimization:

Cycle time : 8		Labor cost : 2.4 ct_1		
operator #	Assignment by machine			Required skills level
	M_1	M_2	M_3	
1	x[1]		x	2
2		x	x	2
3				0

Table 2: results obtained for human resources optimization (job shop 1)

Scheduling and transportation resources optimization (see table 2)

Regarding human resources optimization, we notice that the number of required operators is less than the number of machines. In this configuration, two polyvalent operators with a level of skills equal to two are enough to reach the desired cycle time.

Based on the results obtained after applying the first mathematical model, we determine operations scheduling while maintaining a level of WIP as low as possible. Indeed, using the second model in the case of job shop 1, we obtain an optimal schedule illustrated in Gantt diagram given in figure 1. Through this figure, in each diagram, the sequencing is shown at three different levels: process level (upper section), machines (middle section) and operations (lower section).

[1] Cells with "x" symbol show tasks operators are assigned to

Concerning process P_i, the operations are represented on as many lines as WIP needed by process. WIP are designated by EC_s. The WIP defines the number of transportation resources needed to achieve the requested cycle time. One can see that a WIP equal to 4 is needed for a cycle time CT=8.

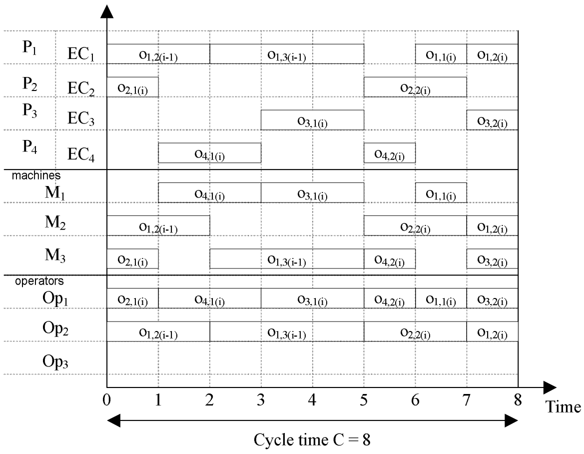

Figure 1: An example of cyclic scheduling for operations of job shop 1

In order to summarize results for job shop 1, we illustrate them through an event graph (figure 2). In this graph, through the control circuits shown in dotted line, we clearly visualize the number of operators (tokens) used and the order in which they perform their respective tasks. The number and the distribution of transportation resources across process circuits are also given.

Furthermore, two more examples are considered.

4.2 Example of job shop inspired from Proth, Sauer and Xie (1997)

Let us consider a job shop (job shop 2) running at minimal cycle time (20 minutes) with four machines and three types of products. The different routings are as follows:

- $Prod_1$: M_1(6), M_2(8), M_3(10), M_4(5) ;
- $Prod_2$: M_4(10), M_3(4) ;
- $Prod_3$: M_4(10), M_3(4).

Production ratios are 25% type products $Prod_1$, 25% type products $Prod_2$, 50% type products $Prod_3$.

Human resources optimization:

Cycle time : 20		Labor cost : 4 ct_1			
operator #	Assignment by machine				Required skills level
	M_1	M_2	M_3	M_4	
1			x		1
2				x	1
3	x				1
4		x			1

Table 3: Results obtained for human resources optimization (job shop 2)

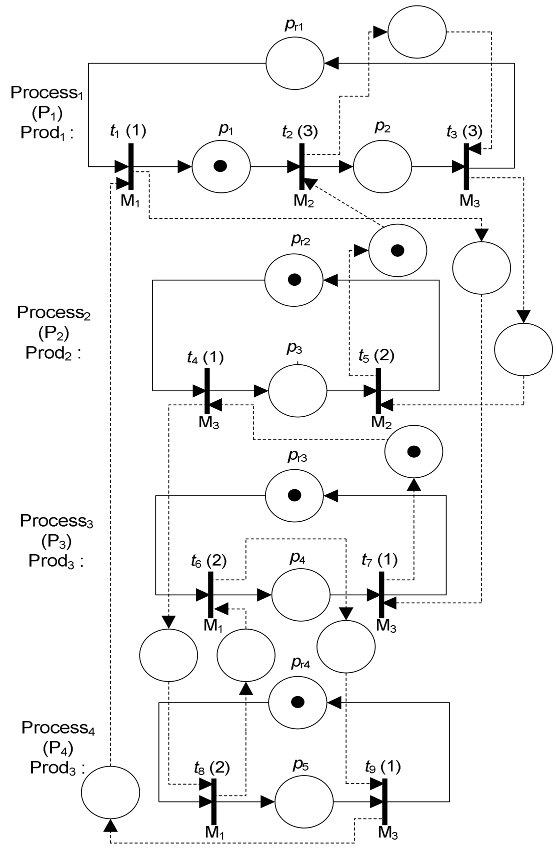

Figure 2: Event graph model of scheduling illustrated in figure 1

Scheduling and transportation resources optimization (see figure 3)

In figure 3, it is obvious that for minimal cycle time there is almost no resources utilization flexibility. Each operator is assigned and qualified to one machine.

4.3 Another example of job shop

Let us consider a job shop (job shop 3) running at a desired cycle time of 15 minutes with five machines and three types of products. The different routings are:

- $Prod_1$: $M_3(5)$, $M_1(3)$, $M_5(4)$, $M_2(2)$;
- $Prod_2$: $M_5(4)$, $M_2(9)$, $M_3(3)$;
- $Prod_3$: $M_4(7)$, $M_2(4)$, $M_1(6)$, $M_3(3)$, $M_5(2)$.

Production ratios are 1/3 products type $Prod_1$, 1/3 products type $Prod_2$, 1/3 products type $Prod_3$.

Human resources optimization

Cycle time : 15	Labor cost : $4.4ct_1$					
operator #	Assignment by machine					Required skills level
	M_1	M_2	M_3	M_4	M_5	
1			x		x	2
2			x	x		2
3	x					1
4		x				1
5						0

Table 4: Results obtained for human resources optimization (job shop 3)

Scheduling and transportation resources optimization (see figure 4**Figure*****)***

Concerning job shop 3, through the results, we observe a particularity. Even with the minimum cycle time, the minimum required number of operators may be less than the number of machines. This becomes possible thanks to a particularly uneven distribution of loads on the machines which allows compensation of one operator removal. In other words, a great value of standard deviation of machines loads allows allotting idle time intervals of operators to other machines.

Through the schedule shown in figure 4, we notice that even with a number of assigned operators (4) less than the number of machines only one operator is saturated. In such a case, an assignment of one operator by machine would be inefficient in terms of resources utilization since there would be significant gaps among the operators' assignments times. As a consequence, that example shows that benefits of polyvalent operators.

4.4 Sensitivity analysis

In previous examples, our model is tested in specific conditions. We aim to apply some flexibility to parameters in order to assess our model performance. Thus, we make a sensitivity analysis for each one of the job shops already taken into consideration.

4.4.1 Scheduling and resources optimization of Job shop 1

We observe in table 5 that the minimum number of required operators decreases as the cycle time increases. In some cycle time ranges (ranges [8, 10 [; [10, 16 [; for example), the number of operators remains unchanged while the cost amount varies. This is explained by the labor costs scale defined for different skill levels. In our case, the growth rate in wage costs tends to decline with increasing skills levels.

Cycle time range	[6, 8[	[8, 10[	[10, 16[	[16, ∞[
Minimum required number of operators	3	2	2	1
Labor cost (factor of ct_1)	3	2.4	2.2	1.4

Table 5: Labor costs and required number of operators for different cycle times

Figure 5 illustrates the variation of the WIP for different values of cycle time. We observe that when the cycle time is increased from 6 to 8, the WIP decreases from 5 to 4 while the number of assigned operators is reduced by 1.

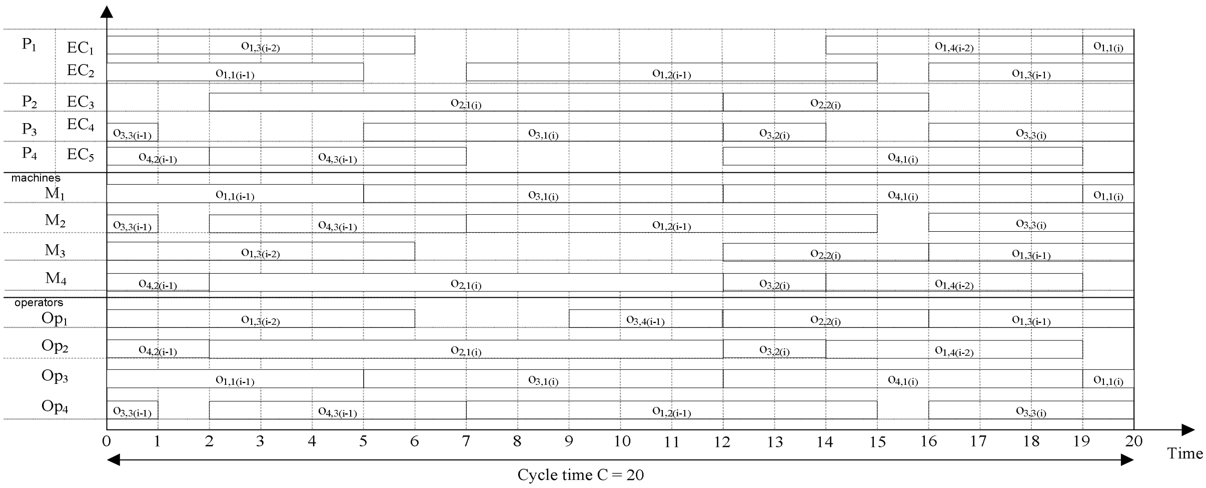

Figure 3: An example of cyclic scheduling for job shop 2

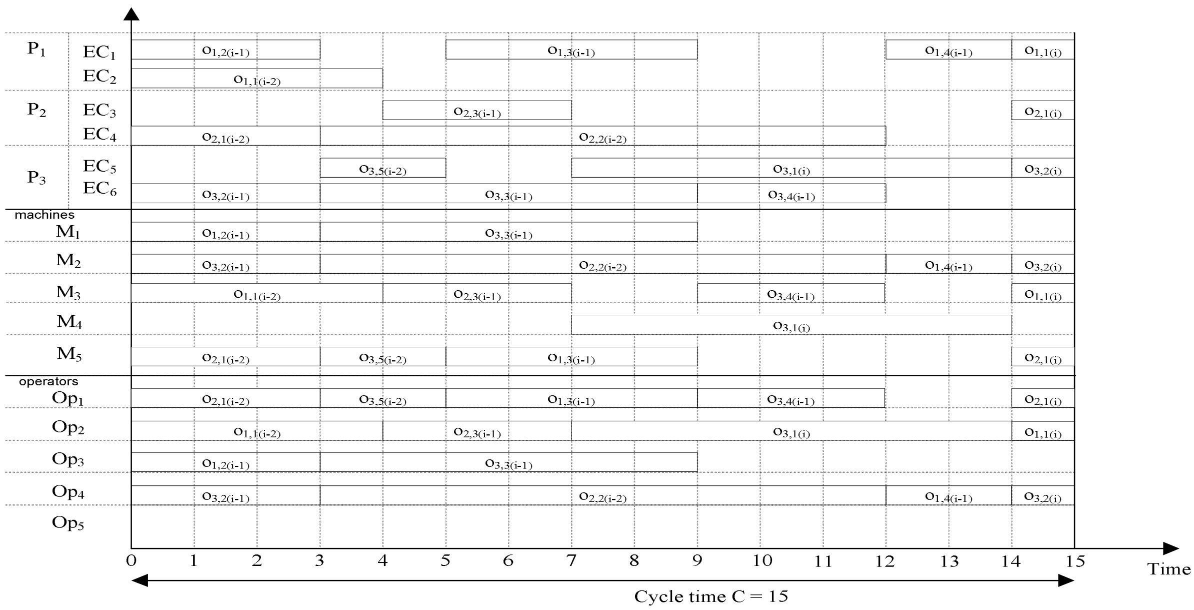

Figure 4: An example of cyclic scheduling for job shop 3

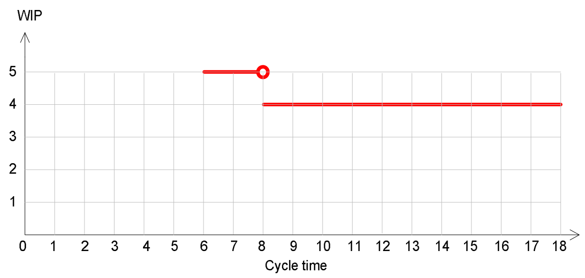

Figure 5 : WIP evolution curve

4.4.2 Scheduling and resources optimization of Job shop 2

Table 6, shows a more important variation of the labor cost. This is the result of the fact that more combinations are possible to assign operators when the number of machines and the number of tasks are higher. We also notice that, with the same number of operators, there are different costs in some cycle time ranges. That is possible by varying operators' qualifications. For example, in table 7, we illustrate two cases of assignment. We choose two values for the cycle time: 36 and 38 minutes. In the first case, the cycle time is 36 minutes and the labor cost of 2.6 times ct_1. This performance is obtained with two operators: one assigned to three machines (M_2, M_3, M_4) and the other one to two machines (M_1, M_4).
The second case is characterized by a cycle time of 38 minutes at the labor cost is reduced to 2.4 ct_1 . Two operators are needed. Each of them has a level of skills equal to 2, i.e. each one is assigned to two machines.

4.4.3 Scheduling and resources optimization of Job shop 3

Trough table 8, we observe that the number of operators and labor cost decrease more or less regularly with cycle time increase.

Cycle time range	[20, 24[	[24, 26[	[26, 32[	[32, 36[	[36, 37[	[37, 71[	[71, ∞[
Minimum required number of operators	4	3	3	3	2	2	1
Labor cost (factor of ct_1)	4	3.6	3.4	3.2	2.6	2.4	1.55

Table 6 : Labor costs and minimum numbers of required operator for different cycle times

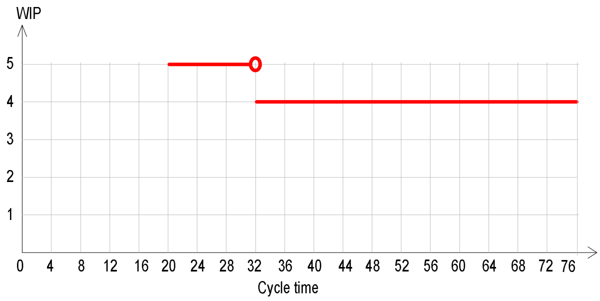

Figure 7: WIP evolution curve

Cycle time : 36					Labor cost : 2.6 ct_1					Average number of operators / machine : 0.5						
operator #	Assignment by machine				Assignment by task *j* of process *i* (*Oij*)												Required skills level
	M1	*M2*	*M3*	*M4*	*O11*	*O12*	*O13*	*O14*	*O21*	*O22*	*O31*	*O32*	*O33*	*O41*	*O42*	*O43*	
1																	0
2	x			x	x			x	x		x			x			2
3																	0
4		x	x	x		x	x			x		x	x		x	x	3

Cycle time : 38					Labor cost : 2.4 ct_1					Average number of operators / machine : 0.5						
operator #	Assignment by machine				Assignment by task *j* of process *i* (*Oij*)												Required skills level
	M1	*M2*	*M3*	*M4*	*O11*	*O12*	*O13*	*O14*	*O21*	*O22*	*O31*	*O32*	*O33*	*O41*	*O42*	*O43*	
1																	0
2	x	x			x	x					x		x	x		x	2
3																	0
4			x	x			x	x	x	x		x			x		2

Table 7: Assignment matrices (job shop 2)

Cycle time range	[15, 16[	[16, 18[	[18, 19[	[19, 26[	[26, 37[	[37, 52[	[52, ∞[
Minimum number of required operators	4	4	3	3	2	2	1
Labor cost (factor of ct_1)	4.4	4.2	3.6	3.4	2.6	2.55	1.7

Table 8: Labor costs and minimum numbers of required operator for different cycle times

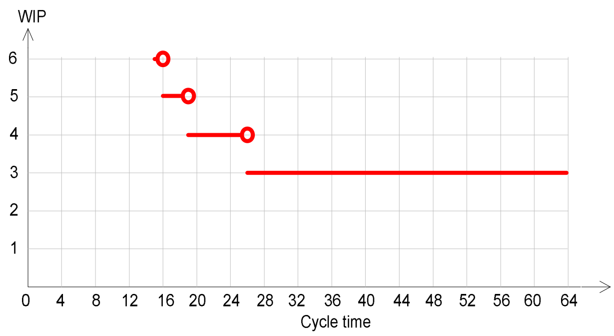

Figure 7: WIP evolution curve

The results regarding WIP minimization can be observed through the curve in Figure 7. It appears that for relatively small variations of the cycle time in the interval [15, 26], the WIP decreases three times, form 6 to 3 units.

4.5 ANALYSIS

The results obtained show that in FMS optimization, labor cost can be efficiently managed by paying a special attention to operator qualifications. The examples that we considered that beyond the flexibility introduced by polyvalent operators, choosing the right qualifications for a company's manpower may be an important factor to reduce production costs.

We also found out that labor cost structures plays an important role and it seems to be the best way to stimulate operator's polyvalence.

It turned out that the human resources cost almost always decreases with the ratio of average number of operators per machine which is illustrated by table 5. Hence, polyvalence is an effective optimizing factor for labor cost.

However, it should be noted that the ratio of average number of operators / machine still does not necessarily explain cost minimization due to polyvalence. Indeed, quite often, the cost structure is not linear. So, depending on salary scale, for two different cycle times, disproportional tasks distribution to operators may be privileged while keeping the same ratio operators / machine. This situation is illustrated in table 7.
Furthermore, increasing cycle time helps reduce labor cost while keeping a level of productivity close to optimum (see table 8). Labor costs minimization may be obtained by a reasonable combination of manpower polyvalence and increased cycle time. Sometimes, cycle time increase is not required to begin getting that cost reduction (see results for job shop 3 at minimal cycle time).
Generally, the solutions found by our method achieve both human and transportation resources optimization. However one may notice than in job shop 3 example, we there are less operators than machines but more WIP (6) than processes (5). This situation is due to the great variance of machines workloads. In this case, resources sharing constraints are reinforced by the limited operators' availability for machine in a critical period of time. It becomes practically necessary to compensate with the transportation resources.

5 CONCLUSION

In this paper, we addressed the problem of resources optimization a FMS. We considered this problem in terms of human resources sharing between multiple machines and focused on a minimization approach of those resources cost based on skills level. The solution we propose to this problem is based on a mixed linear programming approach which allows determining assignments of the minimum required number of operators for different productivity levels. Based on solutions obtained from the first model, we used a method of cyclic scheduling to minimize transportation resources.

Numerical applications highlighted that operators' polyvalence is a decisive factor for labor cost reduction and subsequently overall expenses. Although, sometimes this polyvalence induces an additional transportation resources cost. A slight increase of cycle time may lead to interesting solutions both for labor and WIP costs.

However, the method is limited given the relative narrowness of its scope. It would be interesting to relax the hypothesis of having one operator for each control circuit, to consider standard transportation resources (shared between processes) and even stochastic operating times.

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