AIR TRAFFIC MANAGEMENT USING PETRI NET SYNTHESIS TOOLS

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ABSTRACT: An air traffic system can be considered as a timed discrete events system. Based on the initial flight plan, a synthesis method is used to manage the air traffic. By minimizing a cost function an optimal flight plan is generated which guarantees the respect of specifications in terms of space temporal constraints engendered by adverse weather. The used approach is based on a Timed Petri Net model followed by a Time Reachability Graph analyses.

KEYWORDS: Air traffic, flight, cost, optimization, time Petri Net, time reachability graph.

1. INTRODUCTION

Poor weather conditions leads to a reduction of visibility. To ensure the safety of traffic, the controller requires a larger separation distance between the aircraft. Increase in traffic and adverse weather conditions are among the main factors of congestion in the system of airspace. This congestion causes an increase in the duration of flight delays in the air and on the ground that sometimes induces flight cancellation. Studies done by the Federal Aviation Administration (FAA) have shown that the number of flight delays totaled 900,000 and 93,000 cancellations occurred in 2007 in the United States, according to a report by the Ministry of Transport.

This delay is costly for the airline companies which have resulted in the loss of billions of dollars each year. The cost of fuel consumed by the aircraft in the air represents the highest percentage of the total cost of flight, and since this cost will be added to other costs it is better that the airplane does not delay in air. Furthermore, delays on the ground represent an indirect cost as the customer is not satisfied, which can lead to a loss of business. So air traffic management (ATM) has become a necessity to solve the problem of use of air traffic by reducing the delay cost.

We seek to present in this paper a formal method for the management of air traffic in the airspace which is subject to weather conditions, in addition to an optimization cost in terms of flight delay on the ground and the amount of fuel consumed in the air.

In (Bertsimas and Stock-Patterson, 1998) and (Nilim, Elghaoui and Duong, 2002) determine how to deflect the plane in the system of air traffic control with dynamical weather conditions. The objective of this problem is the minimization of the total delay cost; these studies are based on a deterministic environment, a number of sectors and airports and determined over a period of simulation known. Richetta and Odoni, 1993 and Wang and Zhang, 2005, first proposed an integer programming model to solve the multi-period single airport static stochastic ground holding problem. In their model, uncertainty in airport capacity is represented by a finite set of scenarios, each one represents a time varying profile of the airport capacity that is likely to occur. More recently, Kotnyek and Richetta, 2006, showed that when the ground holding costs are marginally increasing the dual of the model presented in (Richetta and Odoni, 1993) becomes a minimum cost network flow problem in formulation, and hence integer solutions are guaranteed to be obtained by solving the Linear Programming relaxation of the model.

In Airbus-a- authors present the cost index as a tool that allows the airline Airbus to focus on duration of flights, or to minimize the amount of fuel consumed, depending on the conditions of operation they wish to use, and in this study focus
on minimizing the total cost of flights to determine the model used by the digital computer onboard the aircraft (FMS).

The cost from customer dissatisfaction is present both in delays that take place on the ground and delays in the air. Combining these different types of costs gives a positive cost for each time period that a flight is forced to wait on the ground, and an even greater cost for each time period that a flight must wait in the air.

An air traffic system can be considered as a timed discrete event system while its associated management problem can be solved by the synthesis methods which we previously examined (Ghaffari and all, 2003), (Achour and all, 2004) and (Achour and all, 207). Furthermore, the aim of this work is to propose a method for managing air traffic which yields an optimal flight plan which in turn guarantees the respect of specifications in terms of temporal space constraints via the minimization of a cost function. Based on the initial road map and taking into account the uncontrollable events (adverse weather, the delay due to breakdowns on the ground some aircraft of ...), we apply the supervisory control on the management of air traffic so as to determine all acceptable flight plans and identify the optimal one. This problem is dealt with according to formalization in stochastic programming integrating the climatic constraints (Toktas, 2003) and (Toktas , Yen J. W., Zabinsky Z. B., 2006).

2. THE AIR TRAFFIC MANAGEMENT PROBLEM

The air navigation system consists of aircraft, airports, airways and airspace. Airspace is divided into sectors, each sector is managed by two or three controllers. Each sector has a capacity threshold that a maximum number of aircraft that can fly over the same time in normal weather conditions with a distance of horizontal separation standard between 5NM and 8NM (1NM = 1.852Km).

Each aircraft has its own flight plan before take-off from the airport of departure. A flight plan is the set of instructions on the scheduled date of departure, air routes, the sectors that must be past and other information related to characteristics of the aircraft (speed, flight altitude, the number of staff captain). Unfavorable weather has reduced the ability of sectors such as airspace; therefore changing the flight plan has become necessary.

**Example 1.** The air traffic system depicted in figure 1 comprises two planes a and b taking off respectively from airports A and B to land at airport C. Suppose that in their initial flight plans, both planes take off at the same time and fly over the same sectors S1 and S2. Suppose also that adverse weather affecting sector S1 reduces its capacity to only one plane. New flight plans must hence be generated so as to respect the space-time constraint related to the sector capacity. In this regard here are some of the alternatives that can be envisaged:

- Both planes take off at the same time but plane a is rerouted on sectors S3 and S2 in order to avoid S1;
- Both planes take off at the same time but plane b is rerouted on sectors S4 and S5 in order to avoid S1;
- Plane a is delayed until plane b traverses sector S1;
- Plane b is delayed until plane a traverses sector S1.

![Figure 1: Air traffic example](image_url)
any specific cost function but makes the assumption that it is increasing in the amount of delay.

3. OVERVIEW OF SYNTHESIS TOOLS

This section introduces some models and tools we previously developed for the supervisory control of discrete event systems. These will then be applied in the next section to the air traffic management problem at hand.

3.1. Time Petri Net

This section presents some definitions used in this paper. A Timed Petri Net is a tuple N=(P, T, Pre, Post, Is), where P is a set of places; T is a set of transitions; Pre: P x T → N is the pre incidence function that defines weighted arcs from places to transitions; Post: P x T → N is the post incidence function that defines weighted arcs from transitions to places; N is the set of non negative integers and Is: T→N×N is a function which associates a time interval to each transition. For ti∈T, Is(ti)=[ai, bi] such that ai, bi ∈ N and ai ≤ bi ; 0 ≤ ai < ∞ ; 0 ≤ bi < ∞.

The set of input (resp. output) transitions of a place p is denoted by •p (resp. p•). Similarly, the set of input (resp. output) places of a transition t is denoted by •t (resp. t•).

The following Petri net notation will be used throughout the paper:
- M: marking (M0 is the initial marking),
- M(t) = M∪M'; transition t is firable at M (leading to M'). This notation is extended to sequence σ of transitions with M(σ) and M(σ)M',
- σ: occurrence vector whose ith entry denotes the number of occurrences of transition ti

A marked TPN is defined by the couple (N, M0), where, N is a TPN and M0 is the initial marking. In the rest of the paper, when no confusion is possible, a marked TPN is called TPN. A Petri net can also be represented by the incidence matrix C defined as C(piti) =Post(piti)-Pre(piti). A transition tij is enabled by a marking M if and only if M(piti)≥ Pre(pi, tij), ∀ piti∈t•i. This will be denoted by M[tij]. An enabled transition can be fired if the associated time condition is satisfied. The time condition Is(tij)=[ai, bi] is satisfied if the amount of time elapsed since the last enabling time of tij belongs to the time interval [ai, bi]. An example of TPN is given in figure 2.

Initially, places p1 and p2 are marked. The enabled transitions are t1 and t2. For instance, M0(t1|M1, t1 can be fired from M0 if an amount of time between 0 and 2 time units has elapsed since its last enabling moment.

Firing a transition tij from a marking M yields a new marking M' such that M'=M+C(., tij). A marking M' is reachable from a marking M if there exists a firing transition sequence σ= t1 t2 t3…tn transforming M into M'. This evolution is denoted by M[σ→M']. The set of all reachable markings from M0 in G is denoted by R(G, M0). Any marking M reachable from M0 by firing transitions sequence σ satisfies the state equation: M=M0+C(σ), where C: T→N is a vector of non-negative integers, called the occurrence vector. The jth entry in this vector corresponds to the number of times transition tij has been fired in σ.

To each transition tij is associated a clock τij. It counts the time elapsed since the last enabling moment of tij in order to check if the associated time condition is satisfied.

3.2. Time Reachability Graph

The behavior of a TPN is modeled by a Time Reachability Graph (TRG). A TRG is a tuple G=(X, x, Σ, X0, τ), where:
- X is the set of macro-states;
- x= Σx, where x∈X is the set of timed micro-states belonging to a macro-state X;
- ΣΣ=Σ∪Σ is the set of events, where ΣΣ (resp. Σu) is the set of controllable (resp. uncontrollable) events;
- X0 is the initial macro-state;
- τ denotes the time event.

Each node of the TRG, called a macro-state, represents a particular marking of the TPN. A macro-state Xi corresponds to a TPN marking M0. Each macro-state Xi contains a set of timed micro-states xik which memorizes the evolution of the system due to the elapsing of k time units inside the macro-state Xi. Each timed micro-state xik corresponds to a particular value of the clock associated to each transition enabled by the marking M0.

For example, figure 3 shows a TRG modeling the behavior of the TPN of figure 2. The macro-state X0 models the marking M0 where the enabled transitions are t1 and t2. Both transitions are enabled simultaneously. Transition t1 can be fired between 0 and 2 time units, while, transition t2 can be fired between 2 and 3 time units. Therefore, the macro-state X0 contains three timed micro-states: (0 0), (1 1) and (2 2), according to the value of clocks τt1 and τt2. Firing the transition t1 from the macro-state X0 (marking M0) leads to a new macro-state Xi.
Let us suppose that \( t_1 \) is fired from the timed micro-state \( x_{01} = (1, 1) \). When the plant reaches the macro-state \( X_1 \), the clock \( \tau_t \) related to the transition \( t_2 \) has the value 1. Therefore, the system reaches the timed micro-state \( (1) \). The evolution inside each macro-state is generated by the elapsing of time, while the evolution between macro-states is generated by TPN transition firing.

4. Application of synthesis tools to the air traffic management problem

An air traffic system can be considered as a timed discrete events system. A synthesis method is used to manage the air traffic and propose flight plans which guarantee the respect of specifications in terms of space temporal constraints engendered by the adverse weather by means of minimizing a cost function. This method is based on a Timed Petri Net model followed by a search of the Time Reachability Graph.

4.1. Timed Petri Net Model

Let us again consider the air traffic system shown in figure 1. The behavior of each flight is modeled by a Timed Petri Net. The Petri Net places represent airports and sectors. When the transitions represent the passage from one sector to another, the time interval corresponds to the time needed to do so. If, a place represents an airport, its output transition and time interval respectively correspond to the takeoff of an aircraft and the moment of the aircraft takeoff. The transition firing at the moment 0 means that the aircraft takes off without delay. The interval’s upper limit represents the maximum delay which the aircraft can tolerate before the flight canceled.

The aircraft behavior of example 1 is illustrated in figure 4. The flight route of aircraft \( a \) is represented by one of the following two sequences: \{ (departure airport A, S1 sector, S2 sector, arrival airport C) or (departure airport A, S3 sector, S2 sector, arrival airport C) \}. The flight route of aircraft \( b \) is either \{ (departure airport B, S1 sector, S2 sector, arrival airport C) or (departure airport B, S4 sector, S5 sector, arrival airport C) \}. The time interval \([0, 2]\) related to transitions \( t_1 \) and \( t_3 \) indicates that aircraft \( a \) may take off at most two time periods late. After that, the flight will be cancelled. An aircraft needs two time periods to fly through sector S1 \([2, 2]\) related to \( t_4 \), one time period to fly through S2 \([1, 1]\) related to \( t_5 \) and three time slices to fly through S3 \([3, 3]\) related to \( t_2 \).

4.2. Search of the Time Reachability Graph

We proceed as follow to minimize the cost

\[ (1) \]

\[
\text{Figure 3: TRG of the TPN presented in figure 3.}
\]

\[
\text{Figure 4: Time Petri Net modeling.}
\]

Note that air delay is not considered in this example but could be added by changing the interval’s upper limit. The air delay cost is usually larger than ground delay cost, it’s better to adjust the ground delay and minimize the air delay.
function in order to generate the optimal flight plan. The idea consists in memorizing the solution with the least cost yet achieved during the search of the Time Reachability Graph, and comparing it to the cost of each traversed node. If the considered micro-states cost is higher than the least cost to date, the search of the branch is stopped. The search is also stopped if a state violates the capacity constraint. The final least cost solution represents the optimal flight plan.

![Figure 5: Search of the Time Reachability Graph.](image)

So from TRG, we will associate with each micro-states the cost is based on the cost function being established and which we will explain in the next paragraph. So from the micro-state could be in real time the cost of flight time each phase of flight.

A part of the Time Reachability Graph related to the Timed Petri Net of example 1 is presented in figure 5. This graph contains an initial macro-state where the aircraft are at their respective initial airports. This macro-state contains three micro-states [0, 0], [1, 1] and [2, 2]. Each micro-state represents one time slot ground delay. For each micro-state, delay cost is computed (figure 7).

![Figure 6: Cost associated to each micro-state.](image)
This delay cost, corresponds to the cost of shipboard personnel, the maintenance costs and the ground delay cost. These costs will be applied in the following paragraph.

Firing $t_a$ from the macro-state $P_4P_6$, leads to a forbidden macro-state where the aircraft are in the affected sector $S_7$. The macro-state $P_5P_9$ is the final macro-state and represents the aircraft at the destination airport. From the least final micro-state cost, backtracking is done up to the initial micro-state. This path indicates the optimal flight plan.

5. Function cost model

Consider a set of flights $F=\{f_a, f_b\}$, a set of airports $\{A, B, C\}$ and a set of time periods $T=\{15\text{min}, 30\text{min}, 45\text{min}, 60\text{min}, 75\text{min}, 90\text{min}\}$.

\[
C = \sum_{f \in F} [C^g_f + C^T_f + C^k + C_c + C_r]
\]  

(1)

With:

- $C^g_f$: total ground delay cost;
- $C^T_f$: total cost in terms time ;
- $C^k$: fuel consumed cost;
- $C_c$: fixed cost independent of time;
- $C_r$: road cost.

Thereafter we will detail the various costs.
- The total delay cost on the ground $C^g_f$ includes the royalty of car park $C^d_g\delta t^g$ and the cost of passengers delay as shows $C^d_p\delta t^g$ the equation:

\[
C^g_f = (C^d_g + C^d_p)\delta t^g
\]  

(2)

The parking fine per time unit $(C^d_g)$ is a cost paid by the airline at the airport, this cost is paid by the company before the flight, it varies from one airport to another and the duration of the crossing of what accounted for the cost in general, is between 1h15min and 2h.

The delay cost of passengers per time unit $C^d_p$ is equal to:

\[
C^d_p = C^d_p \times N_p
\]  

(3)

$C^d_p$ denote the delay cost per passenger per time unit multiplied by the number of passengers $N_p$. This cost is covered by the airline company. This compensation can be in many forms; meals tickets, hotel accommodation if it is necessary...

$C^T_f$ denote the cost in terms of time per time unit, it includes the crew cost per time unit $C_p$ along with the maintenance cost per time unit $C_M$ as shown in equation 10.

\[
C^T_f = C_p + C_M
\]  

(4)

The cost of kerosene consumed $C^k$ is varied from one airport to another, before takeoff and for the airline which minimize the maximum cost of the flight. It is necessary to study the difference in the cost of kerosene from the departure airport to the destination airport.

The quantity of fuel consumed per time unit $\Delta k^f$ in the phase-off is almost one and a half times the amount which is consumed in a cruise $\Delta k^c$ thus it is essential for the transformation of $\Delta k$ transformed as follows:

\[
\Delta k = \Delta k^f \times \delta t^f + \Delta k^c \times \Delta t^c
\]  

(5)

With

- $\delta t^f$: The time required for the take-off phase;
- $\Delta t^c$: The time of the cruise phase.

The fixed cost is $C_c$ includes landing fee $(R_1)$ and the fee markup $(R_b)$ as shown in equation 10.

\[
C_c = R_1 + R_b
\]  

(6)

For $R_a$ landing fee is paid by the airline at the airport against the use of the infrastructure of the airport on his landing and takeoff.

This fee is depends on the maximum take-off (MMT) of the aircraft during its take-off is related to the capacity of the aircraft, which equals:

\[
R_1 = t \times N
\]  

(7)

With

- $t$: Rate unit fee.

This rate varies from year to another as determined by the Eurocontrol

\[
N = 1.274 \times (\text{MMT})^{0.9}
\]  

(8)

Thus the royalty of landing is equal:

\[
R_a = t \times 1.274 \times (\text{MMT})^{0.9}
\]  

(9)
For \( R_{\text{F}} \), there is a landing fee paid by the airline company to airport, as well as a cost incurred for the ignition lighting which is necessary for each landing and takeoff. Therefore, our overall cost function is as follows:

\[
C = \sum_{v \in V} \left[ (C^{d^g} + C^{d^p})\delta t^g + C^T \Delta t + C^k (\Delta k^t \times \delta t^t + \Delta k^c \times \Delta t^c) + C_c + C_r \right]
\]  

(10)

6. Digital Application

It is based on the map of air traffic management network in France, taking the distances traveled by flights \( a \) and \( b \) in nautical miles (NM) in the various sectors, which shows the following:

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_a )</td>
<td>427.5</td>
<td>213.75</td>
<td>641.25</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( V_b )</td>
<td>427.5</td>
<td>213.75</td>
<td>-</td>
<td>641.25</td>
<td>213.75</td>
</tr>
</tbody>
</table>

Table 1: Distance crossed in every sector.

In this study we will work on the Airbus 320. The following table summarizes the characteristics of A320 which are necessary in our calculation, Airbus \( a \) and Airbus \( b \).

<table>
<thead>
<tr>
<th></th>
<th>( C_M ) $/\text{mn}$</th>
<th>( C_p ) $/\text{mn}$</th>
<th>( \Delta k^d ) $/\text{kg/mn}$</th>
<th>( \Delta k^c ) $/\text{kg/mn}$</th>
<th>( V_c ) $/\text{km/mn}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A320</td>
<td>3 $&lt; 7$</td>
<td>5 $&lt; 10$</td>
<td>63</td>
<td>40</td>
<td>14.25</td>
</tr>
</tbody>
</table>

Table 2: The characteristics of A320.

\( V_c \): Cruising speed.

Thus out of the A320, we can determine the time required for the aircraft to move from each sector and the various costs involved for each unit of time.

For the time between the different sectors are calculated using the cruise speed, we find that the A320 aircraft requires 15 min to move from sector \( S_2 \) and \( S_3 \), 30 min from \( S_4 \) and 45 min from \( S_3 \) and \( S_4 \).

In our calculation we took the cost of parking:

\[
C^d = 1.2 \epsilon/\text{tone/u.t}
\]  

(11)

And as the two units of delay time on the ground plans of two flights of no more than 1h30min, so the calculation we will assume that the cost of parking is calculated from \( \delta t^d = 0 \).

For the cost of maintenance personnel and represents the cost in terms of time per time unit we took the average value.

So:

\[
C^T = 138.15 \epsilon/\text{u.t}
\]  

(12)

The cost of kerosene varies from one airport to another and to other global economic factors. As calculated on the assumption that the cost of kerosene in the airport of departure \( (a \) and \( b \) \)  

\[
C^k = 0.5 \epsilon/\text{kg}
\]

which is much less compared to the airport of arrival \( c \) so to minimize the cost of flight must fill the tank sufficiently for the return of the aircraft, therefore the maximum take-off of the aircraft taking into account the extra fuel was noted in the table 3.

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Cr )</td>
<td>327.2</td>
<td>163.6</td>
<td>490.81</td>
<td>490.81</td>
<td>163.6</td>
</tr>
</tbody>
</table>

Table 3: Cost of road paid in every sector.

In this section, we will calculate the cost of each flight plan is associated with each micro-state of accessibility graph corresponds to the cost each time to determine the optimal flight-term cost that meets the objectives of the airline.

This is flight plan corresponds to the passage of two planes at the same time in sector \( S_1 \) in normal weather conditions. In this flight plan the two aircraft made off without delay on the ground, cutting the sector \( S_1 \) and \( S_2 \) arriving at the airport \( c \).
The total cost is the macro-state $P_5P_6$ is $C = 610,655$, this cost represents the cost of fuel consumed and the cost in terms of time during the take-off phase which lasts one time unit, the cost is added to each time unit cost in term time and cost of fuel consumed in the cruise phase. Each time the aircraft passed through an area a cost of road is added to the cost of flight, it is presented for example in the cost corresponds to the micro-state $P_3P_7$, which indicates that the aircraft entered the sector after $S_2$ its transition from sector $S_1$, the cost (C = 3401.1 €) includes the various costs up to this stage of flight the cost of road provides the controller placed in the $S_1$ (327.2€).

The fixed cost includes the cost of lighting and landing is added to the cost of flight when the plane is landing at the airport in $C$ warned that the cost is the macro-state $P_3P_5$, this cost in the fixed cost of flight a. Suppose that time adversely affecting the sector $S_1$ reduces its ability to only one plane. New flight plans must then be produced to meet the time constraint related to space segment capacity.
In Figure 7, we calculate the flight plan in which it has delayed the plane $b$ until the aircraft $a$ crosses the sector $S_1$.

![Diagram of flight plans](image)

Figure 8. Cost Graph

The difference between the total cost of the initial flight plan and there is the cost spent in delay of the aircraft $b$ ground, this cost represents the cost of parking and the cost in term time for two time unit delay.

In Figure 8.b, we calculate the flight plan in which the two aircraft took off together but the aircraft $b$ was diverted to sectors $S_4$ and $S_5$ to avoid $S_1$.

This increase in the cost of the flight plan in relation to the original flight plan and flight plan or delay on the flight plans of $b$ is explained primarily by the amount of fuel consumed by the aircraft $b$ during its passing through the area and $S_1$ by the cost of road paid in this sector, which exceeds the cost paid in $S_2$ because of the distance it longer.
7. Conclusion

This paper deals with a management problem of air, presenting a formal method in an optimal way to adjust flight plans in response to reduced road capacity imposed by the unfavorable time. The cost function is assumed to increase in the amount of delay. The proposed approach is based on a model of Time Petri Net, air navigation system followed by a search of the Graph Accessibility Time. It was associated with each micro-state is the cost for each unit of time phase of flight based on the cost function is established.

The determination of the optimal flight plan is determined from the lowest cost calculated for each flight plan ensuring specifications in terms of temporal space. A cost analysis shows that the calculated cost of fuel is the highest cost paid by the airline, but the additional cost of fuel paid by the airline due to a change in routes original flight plan is better indirect cost accounts for the dissatisfaction of passengers and a subsequent loss of business that affects the reputation of the company and resulting cost lost indefinitely.

ACKNOWLEDGMENTS


