A TACTICAL MODEL UNDER UNCERTAINTY FOR HELICOPTER MAINTENANCE PLANNING

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ABSTRACT: Maintenance is an activity of growing interest in the aeronautical industry. A particularity of this activity is the high level of uncertainty. Consequently, adequate maintenance center management is important to guarantee a good service level, in this case aircraft visit duration, with acceptable costs. The Helimaintenance project focuses on civil helicopter maintenance. This paper presents a tactical planning model for this multiproject activity. It is based on fuzzy sets to represent the uncertainties. We propose a representation of the resource requirements that takes into account these uncertainties.

KEYWORDS: Multiproject planning, uncertainty, fuzzy model, maintenance.

1 Problem statement

Aircraft maintenance consists in carrying out all the actions necessary to guarantee the required level of reliability, safety and operational capacity of the aircraft. A main characteristic of aeronautics is that maintenance is highly regulated, due to the potential criticality of the failures.

From a product point of view, various documents define aircraft maintenance. Among them, the maintenance planning document (MPD), established by the manufacturer on the basis of reliability studies, gives the periodicity of inspection of the equipments (calendar limits and/or number of flight hours and/or number of takeoff-landing flight cycles). It is an evolutive process: the manufacturer periodically updates the documentation (by editing service bulletins). Hence the maintenance tasks may change all along the aircraft lifecycle.

On this basis, the owner of the aircraft establishes the maintenance program, that must be approved by the authorities. The list of the tasks to perform during a maintenance visit depends on the aircraft exploitation and equipment history, with refer to the limits set in the MPD. It also depends on decisions of anticipating some tasks in order to balance the number and the duration of visits i.e. to balance aircraft exploitation and maintenance cost. Finally, unexpected failures may force to anticipate a visit.

From a process point of view, the Aircraft Maintenance Manual describes how to perform the maintenance actions. Regulation also constrains the process: ratio of permanent operators, number of hours by week, skills (European Aviation Safety Agency PART 66 defines the requirements to deliver licenses to the operators, PART 145 for the maintenance center).

Maintenance planning aims at organizing the activity of a maintenance center. It deals with the tasks to realize for each aircraft, the workforce and equipment organization, spare parts purchasing and inventory management. The challenge is to provide to customers minimal maintenance duration, while maintaining good productivity and inventory costs.

The Helimaintenance project focuses on civil helicopter maintenance. In addition to the above considerations, helicopters have some specificities: particular flight conditions (vibrations, way of landing) involve high frequency of inspection for some equipments, limited volume may constrain the realization of tasks (number of operators concurrently working). Addressing civil customers involves a great heterogeneity of the helicopters. Indeed, the mean number of helicopter by civil owner is between 2 and 3, and the conditions of use can radically vary from one customer to another (sea, sand, mountain...). Customization can also lead to very different equipment configurations. Hence the application of global optimization approaches, as can be found in the military domain for important homogeneous fleets and one single customer (Hahn and Newman 2008, Sgaslik 1994), is not pertinent in this case. Next section presents a framework based on multiproject planning to deal with our helicopter maintenance application. Then
we develop the tactical planning level based on a fuzzy model: macro planning (sec. 3) and capacity planning (sec. 4).

2 Multiproject planning for helicopter maintenance

This work focuses on Heavy Maintenance Visits (HMV), the heaviest checks of an aircraft. These checks affect all the aspects (structure, avionics, mechanics) and can last up to several months. On PUMA helicopters for example, HMV is generally done every twelve years and costs about 2 millions Euros (1/3 of the price of the aircraft). The visit contains planned maintenance tasks and also corrective maintenance tasks because problems are discovered during the inspection of the helicopter. Precedence constraints exist, due to technical or accessibility considerations. Hence a HMV may be seen as a project involving various resources as operators, equipment and spare parts. As already noted, minimizing the overall visit duration give a competitive advantage to the company. Consequently, the management of a maintenance center is viewed as multiproject management, where every project duration should be minimized while respecting capacity constraints.

Hans et al. (2007) classify multiproject organizations according to their projects variability and dependency. Following this reference, our problem is considered as high variability (numerous uncertainties) and high dependency (shared resources and external influence on spare parts supply). Then they propose a hierarchical framework for multiproject planning and control. Inspired from manufacturing, this framework is suitable to our problem that gathers production and project features. The framework is divided on the hierarchical axis in the three levels from Anthony’s classification (strategic, tactical and operational) and on the functional axis in technological planning, capacity planning and material coordination. This paper concentrates on the tactical level, on the following functions: (technological) macro task planning (Sec. 3) and rough cut capacity planning (Sec. 4). We consider that spare parts purchasing (material coordination) is out of the scope, and treat this aspect by introducing procurement delays for corrective maintenance tasks.

Dealing with high variability is an important issue. The hierarchical decomposition aims at generating stable plans, coordinating levels and dealing with the adequate horizon and data at each level. At the tactical level, we can identify three main sources of uncertainty:

- Uncertainty on the release date: a customer enters into a contract for a HMV several months before the helicopter entry in maintenance. According to the exploitation, the real start date may vary in order to reach the limits set by the MPD.
- Uncertainty on the workload: the corrective maintenance part is significative in these projects. The nature of the corrective maintenance is only known after the first inspection tasks of the project.
- Uncertainty on procurement delays: though spare parts for planned maintenance can be purchased on time, corrective maintenance induce additional orders. Whether these parts are available in the inventory or should be purchased, or even must be manufactured, the procurement delays may change radically.

To cope with these uncertainties, we propose a combined approach. First, considering the non repetitive aspect of the problem (each helicopter has its own history, the customers are numerous and the conditions of use are very different) and the difficulty to predict the exploitation or establish statistics on corrective tasks, we propose a fuzzy set modeling for project dates and task durations. Second, to limit the range of uncertainty on procurement delays, scenarios are considered, corresponding to the various purchasing options above mentioned.

3 Macro task planning

Macro task planning consists in an infinite capacity planning where tasks are grouped by speciality (structure, mechanics, avionics). For example, a PUMA HMV macro planning contains between 15 and 20 macro tasks. Some of them may be subcontracted. Precedence constraints correspond to the general sequence of the macro tasks (e.g. inspection first, painting at the end...). In the following, projects are considered independantely, even if some obvious capacity constraints as the maximal number of helicopters in the center could already be taken into account. Workforce capacity constraints that create project dependency will be addressed in the rough cut capacity planning.

The planning horizon is set to 12 months in order to cover the overall delay from the contract to the end of the project. This situation can be compared to engineering-to-order (ETO) in manufacturing.

3.1 Fuzzy set modeling

An ensemblist representation can be either a simple interval or a more complex and complete form as triangular or trapezoidal profile (Fig. 1). A profile representation has the advantage to be supported by the possibilistic approach (Sec. 4.1).
Zadeh (1965) has defined a fuzzy set $\tilde{A}$ as a subset of a referential set $X$, whose boundaries are gradual rather than abrupt. Thus, the membership function $\mu_{\tilde{A}}$ of a fuzzy set assigns to each element $x \in X$ its degree of membership $\mu_{\tilde{A}}(x)$ taking values in $[0;1]$.

To generalize some operations from classical logic to fuzzy sets, Zadeh has given the possibility to represent a fuzzy profile by an infinite family of intervals called fuzzy sets, Zadeh has given the possibility to represent classical interval arithmetic and adapt it to fuzzy profiles. Dubois and Prade (1988) and Chen (1992) have defined mathematical operations that can be performed on trapezoidal fuzzy sets. Let $\tilde{A}(a_A, b_A, c_A, d_A)$ and $\tilde{B}(a_B, b_B, c_B, d_B)$ be two trapezoidal fuzzy numbers, then:

\[
\tilde{A} \oplus \tilde{B} = (a_A + a_B, a_B + b_A + b_B, c_A + c_B, d_A + d_B)
\]

\[
\tilde{A} \ominus \tilde{B} = (a_A - a_B, b_A - b_B, c_A - c_B, d_A - d_B)
\]

\[
\min(\tilde{A}, \tilde{B}) = (\min(a_A, a_B), \min(b_A, b_B), \min(c_A, c_B), \min(d_A, d_B))
\]

\[
\max(\tilde{A}, \tilde{B}) = (\max(a_A, a_B), \max(b_A, b_B), \max(c_A, c_B), \max(d_A, d_B))
\]

Other operations like multiplication and division have also been studied. For more details on fuzzy arithmetics we can refer readers to (Dubois and Prade, 1988) and (Neumier, 1990).

### 3.2 Project release date

Figure 2 presents an example of an equipment inspection date determination from helicopter exploitation assumptions, calendar limits and flight hours (top) or flight cycle (bottom) limits, for example 30000 h or 15000 cycles by 10 years. From the update, flight hours evolve in a range going from no exploitation to the physical limits of the aircraft, through pessimistic and optimistic exploitation values. Intersections of these lines with calendar and flight hours limits define the four points $a_H, b_H, c_H$ and $d_H$ of the trapezoidal fuzzy number $\tilde{H}$, inspection date according to the flight hours. It is the same for flight cycles.

For a single equipment, the fuzzy inspection date is the fuzzy minimum of the dates involved by flight hours ($\tilde{H}$) and cycles limits ($\tilde{C}$):

\[
\min(\tilde{H}, \tilde{C}) = (\min(a_H, a_C), \min(b_H, b_C), \min(c_H, c_C), \min(d_H, d_C)).
\]

The fuzzy release date of the project is the fuzzy minimum of the inspection dates of critical equipments listed in the maintenance program, and of the helicopter itself. The uncertainty on this date decreases along the time, as information on actual exploitation increases, so periodic updates should be done.

### 3.3 Macro task durations

Macro task durations are established by asking experts. Rommelfanger (1990) proposes a 6-point fuzzy number to represent the expert knowledge. In this work, however, we will still consider 4-point trapezoidal numbers.

At the tactical level, uncertainty on macro task duration is mainly due to unexpected corrective maintenance. These additional tasks (work and delays) can represent from one third to one half of the total project duration. They generally appear during the structural inspection macro tasks, but the whole project is impacted.

Procurement for corrective maintenance may introduce delays in the planning. As the equipments to be purchased are not known before inspection, we consider scenarios: the equipment is available on site, at an European supplier, at a foreign supplier, or it may be found after some research, or it is obsolete.
and must be manufactured again. According to the information on the helicopter (age of the aircraft, conditions of use, etc.), some scenarios can be discarded from the beginning (e.g. new helicopter ⇒ no obsolescence) and task durations can be refined.

3.4 Fuzzy Critical Path Method

Fortin et al. (2005) describe the algorithms to adapt Critical Path Method to fuzzy numbers. In the deterministic case, the method is based on two successive steps: a forward propagation to determine the earliest start and finish dates (and consequently the project duration and the free floats), and a backward propagation for the latest start and finish dates (and the total floats). In the fuzzy case, forward propagation is done using fuzzy arithmetics, leading to fuzzy earliest dates and a fuzzy end-of-project event. Unfortunately, backward propagation is no longer applicable because uncertainty would be taken into account twice. Fortin et al. (2005) show that the boundaries of latest dates (and consequently floats) are reached in extreme configurations. They propose algorithms to determine these values and show that some problems (e.g. minimal float determination) become NP-hard.

This work only considers earliest dates. Latest dates and floats shall be used later to solve capacity problems. The composition of fuzzy dates and durations results in task fuzzy earliest start and finish dates. These dates are trapezoidal fuzzy numbers.

3.5 Planning update

Updating is done as new information appear. A new planning is established periodically in order to decrease uncertainty on the release date, and at the end of major inspection tasks in order to validate or discard scenarios.

4 Capacity planning

Accounting for resource constraints, even in a deterministic case, leads to difficult problems. Some authors proposed approaches for the Fuzzy Resource Constrained Project Scheduling Problem (Hapke and Slowinski, 1996). Here we propose to establish a resource usage chart based on the fuzzy earliest dates of the project. To this aim, we apply the possibility theory (Zadeh, 1978) in order to represent the presence of a task in a period and deduce the amount of resource required.

4.1 Possibility theory

The possibility theory is based on fuzzy subsets. It was introduced by Zadeh (1978) to provide a mean to take into account the uncertainties in an event occurrence. Since we have chosen a fuzzy model, we will use the possibility theory to transpose date and duration uncertainties on the resource plan. Examples of use of the possibility theory in production planning can be found in (Imiguchi et al., 1994), (Fargier and Thierry, 2000) and (Reynoso, 2004).

The possibility theory introduces both a possibility measure (denoted $\Pi$) and a necessity measure (denoted $\bfN$). Let $P$ be a set (fuzzy or not) and $\tilde{A}$ a fuzzy set attached to a single valued variable $x$. The possibility of the event “$x \in P$”, denoted by $\Pi(x \in P)$, evaluates the extent to which the event is “possibly” true. It is defined as the degree of intersection between $\tilde{A}$ and $P$ by the minimum operation:

$$\Pi(x \in P) = \sup_u \min(\mu_A(u), \mu_P(u)).$$

The dual measure of necessity of the event “$x \in P$”, denoted by $\bfN(x \in P)$, evaluates the extent to which the event is “necessarily” true. It is defined as the degree of the inclusion ($\tilde{A} \subset P$) by the maximum operation:

$$\bfN(x \in P) = \inf_u \max(1-\mu_A(u), \mu_P(u)) = 1-\Pi(x \in P^c)$$

where $P^c$ is the complementary of $P$ ($\mu_{P^c}(u) = 1 - \mu_P(u)$).

Let $t$ be a real-valued variable in the fuzzy interval $\tilde{A}$ and $\tau$ be a constant. To measure the truth of the event $\tilde{A} \supseteq [\tau; \infty)$, we need the couple $\Pi(\tau \leq t)$ and $\bfN(\tau \leq t)$ (Fig. 3). Thus :

$$\Pi(\tau \leq t) = \sup_{u \geq \tau} \mu_A(u) = \mu_{(-\infty, a]}(\tau)$$

$$= \sup_u \min(\mu_A(u), \mu_{[\tau, +\infty)}(u))$$

and

$$\bfN(\tau \leq t) = \inf_{u < \tau} (1-\mu_A(u)) = \mu_{(-\infty, a)}(\tau)$$

$$= \inf_u \max(1-\mu_A(u), \mu_{[\tau, +\infty)}(u))$$

Figure 3: Necessity of $\tau \leq t \in \tilde{A}$.

Let $\tau$ and $\sigma$ two variables in respectively fuzzy intervals $\tilde{A}$ and $\tilde{B}$ and $t$ a real value. To measure the truth of the event “$t$ between $\tau$ and $\sigma$” we need both $\Pi(\tau \leq t \leq \sigma)$ and $\bfN(\tau \leq t \leq \sigma)$ . Thus:

$$\Pi(\tau \leq t \leq \sigma) = \mu_{[\tilde{A}, \tilde{B}]}(t) = \mu_{[\tilde{A}, +\infty) \cap (-\infty, \tilde{B}]}(t)$$

$$= \min(\mu_{[\tilde{A}, +\infty)}(t), \mu_{(-\infty, \tilde{B}]}(t))$$
and
\[ N(\tau \leq t \leq \sigma) = \mu_{\tilde{A},\tilde{B}}(t) = \mu_{\tilde{A},+\infty} \cap (-\infty,\tilde{B}(t)) \]
\[ = \min(\mu_{\tilde{A},+\infty}(t), \mu_{(-\infty,\tilde{B}(t))}(t)) \]

Figure 4 presents the possibility and necessity membership functions for an event \( t \) to be between fuzzy intervals \( A \) and \( B \).

\[ \begin{array}{ll}
1 & \mu_{\tilde{A}} \\
0 & \mu_{\tilde{B}} \\
\end{array} \]
\[ a_A \quad b_A \quad c_A \quad d_A \quad a_B \quad b_B \quad c_B \quad d_B \]

Figure 4: Necessity and possibility of \( t \) being between \( A \) and \( B \).

4.2 Presence of a task

Let \( \tilde{S}(a_S, b_S, c_S, d_S) \) be the fuzzy earliest start date of a task \( T \), \( \tilde{F}(a_F, b_F, c_F, d_F) \) its earliest finish date and \( \tilde{D}(w, x, y, z) \) its duration. Relations between these values are:

\[ a_F = a_S + w \]
\[ b_F = b_S + x \text{ with } w \leq x \leq y \leq z \]
\[ c_F = c_S + y \]
\[ d_F = d_S + z \]

Using fuzzy arithmetics, we can define \( \tilde{S}; \tilde{F} \) (resp. \( [\tilde{S}; \tilde{F}] \)), the domain where the presence of task \( T \) is necessary (resp. possibly) true. They represent the truth of the event “\( t \) between the start and the finish date of \( T \)”. Associated membership functions, \( \mu_{\tilde{S},\tilde{F}}(t) \) and \( \mu_{[\tilde{S}; \tilde{F}]}(t) \) are respectively denoted \( N(t) \) and \( \Pi(t) \) in the following (Fig. 5).

\[ \begin{array}{ll}
1 & N(t) \\
0 & \Pi(t) \\
\end{array} \]
\[ w \quad x \quad y \quad z \]
\[ as \quad bs \quad cs \quad ds \quad aF \quad bF \quad cF \quad dF \]

Figure 5: Presence of a task.

In the configuration of Figure 5, we can identify the following intervals of possibility and necessity:

\[ [d_S; a_F] \quad : \quad \Pi = 1 \quad N = 1 \]
\[ [c_S; d_S] \quad : \quad \Pi = 1 \quad N \geq 0 \]

Then we characterize the probability of presence of task \( T \) as a distribution \( p(t) \) situated between the possibility and the necessity profile: \( N(t) \leq p(t) \leq \Pi(t) \).

We propose a parametric piecewise linear distribution to represent the probability of presence of the task (dashed line on Fig. 5). It will be used to establish resource requirement. This distribution corresponds to the intervals of possibility and necessity:

\[ p(t) = \begin{cases} 
\frac{H}{b_S - a_S}(t - a_S) & \text{if } t \in [a_S; b_S] \\
\frac{1 - H}{d_S - c_S} \left( t + \frac{H d_S - c_S}{1 - H} \right) & \text{if } t \in [c_S; d_S] \\
1 & \text{if } t \in [d_S; a_F] \\
\frac{H - 1}{b_F - a_F} \left( t + \frac{b_F - H a_F}{H - 1} \right) & \text{if } t \in [a_F; b_F] \\
\frac{-H}{d_F - c_F}(t - d_F) & \text{if } t \in [c_F; d_F] \\
0 & \text{otherwise,} 
\end{cases} \]

where parameter \( H \), varying from 0 to 1, makes profile \( p(t) \) evolve from \( N(t) (H = 0) \) to \( \Pi(t) \) (\( H = 1 \)).

Figure 5 presented a configuration without overlap between fuzzy start and finish dates \( (d_S \geq a_F) \). Two more configurations exist (Fig. 6 and Fig. 7): small overlap \( (d_S > a_F \text{ and } c_S \leq b_F) \) and large overlap \( (c_S > b_F) \). Note that the maximal necessity value is respectively lower than 1 and equal to 0 for these configurations. In the same way that for the first configuration, a parametric piecewise linear distribution can be defined to represent the probability of presence of the task.

For the small overlap configuration, the distribution is (dashed line on Fig. 6):

\[ p(t) = \begin{cases} 
\frac{H}{b_S - a_S}(t - a_S) & \text{if } t \in [a_S; b_S] \\
\frac{1 - H}{d_S - c_S} \left( t + \frac{H d_S - c_S}{1 - H} \right) & \text{if } t \in [c_S; a_F] \\
\frac{H - 1}{b_F - a_F} \left( t + \frac{b_F - H a_F}{H - 1} \right) & \text{if } t \in [a_F; b_F] \\
\frac{-H}{d_F - c_F}(t - d_F) & \text{if } t \in [c_F; d_F] \\
0 & \text{otherwise.} 
\end{cases} \]
In this paper we propose to build a task resource usage profile in a way that keeps track of uncertainty on start and finish dates. Hence the profile reflects the whole possible time interval while giving a plausible repartition of the workload according to the duration parameter value. To this aim, the resource usage profiles are defined as a projection of the task presence distributions onto the workload space.

### 4.3 Resource usage profile

The establishment of a relevant resource usage profile for a task with fuzzy dates and duration is difficult. Most of the time, the problem parameters are fixed in order to obtain a deterministic configuration. This leads to a scenario approach (Hapke and Slowinski, 1996) where various significant scenarios may be compared in a decision process: lower and upper bounds, most plausible configuration, etc.

Let us consider the case with no overlap between \( \tilde{S} \) and \( \tilde{F} \) (Fig. 8 top). Suppose that the resource requirement of the task is \( r \). Resource workload then lies in \([r.w; r.z]\), according to the task duration. The idea is to represent the extreme workload profiles by a projection of the presence possibility for maximal workload \( r.z \) and presence necessity for minimal workload \( r.w \) (Fig. 8).

Consequently, for any duration between \( w \) and \( z \), the resource usage profile is represented by a projection of the task presence probability distribution \( p(t) \) introduced in Section 4.2. The link between the task duration and the profile, for a duration \( D \), is given by the following formula:

\[
r.D = \int_{0}^{+\infty} r.p(t)dt = r.H \left( \frac{d_F - a_S}{2} + \frac{c_F - b_S}{2} \right) + r.(1 - H) \left( \frac{a_F - d_S}{2} + \frac{b_F - c_S}{2} \right).
\]

Hence the extreme profiles should correspond to \( H = 0 \) and \( H = 1 \). Unfortunately, the areas of the projections of the necessity and possibility distributions do not generally match with extreme resource consumption \( r.w \) and \( r.z \). If the area of the projected
necessity distribution is smaller than \( r.w \), \( H \) is limited to \( H_{\text{min}} > 0 \) so that \( r.w = \int_0^{+\infty} r.p(t)dt \). In the same way, if the area of the projected possibility distribution is greater than \( r.z \) then \( H \) is limited to \( H_{\text{max}} < 1 \) so that \( r.z = \int_0^{+\infty} r.p(t)dt \). Figure 9 shows an example of modified extreme profiles.

![Figure 9: Resource profiles: restriction to \( H_{\text{min}} \) and \( H_{\text{max}} \) in order to match with extreme workloads \( r.w \) and \( r.z \).](image)

Let us now consider the particular case of a task with a fuzzy duration, but a deterministic start date \( (a_S = b_S = c_S = d_S = s) \), Fig. 10.

![Figure 10: Case of a deterministic start date: presence distributions and maximal resource profile.](image)

If we consider the case where task duration \( D = z \), the associated resource chart is the one of Figure 10 rectangular shaped. Indeed, the start date being deterministic, if we choose \( D = z \), there is only one possible position for the task, between \( s \) and \( d_F \). So the resource chart is fixed. One can remark that in this case, the projection of the possibility distribution (dashed line) is not able to represent the resource consumption: even with \( H = 1 \), the resource workload would be underestimated. Indeed, the area delimited by the dashed line between \( s \) and \( d_F \) is \( r.(c_F - s + d_F - s)/2 = r.(y + z)/2 \). For any duration \( D > (y + z)/2 \), this area is too small to represent the resource workload. To cope with this problem, we modify the resource profile: in place of points \((s, s, c_F, d_F)\), the new profile is defined by the points \((s, s, c_F, d_F)\), where \( c_F' = c_F + \max(0, 2D - z - y) \). Hence, while \( D \leq (y + z)/2 \), initial profile is used and \( H \leq 1 \). Then \( H = 1 \) and until \( D = z \), the new profile is used. When \( D = z \), the initial rectangular profile is attained.

A similar modification can be done for the minimal duration, when the area of the projected necessity distribution is greater than \( r.w \). These modifications can be generalized to the case with fuzzy dates and duration. Then the profiles, if needed, are modified on both sides. For the maximal workload, the area covered by the projection of the possibility distribution is:

\[
r.H = r.\left(D - a_S + c_F - b_S\right)/2.
\]

The extended maximal profile, defined by \((a_S, b_S, c_F, d_F)\), is used when \( D < D \leq z \). Values \( b_S' \) and \( c_F' \) are:

\[
b_S' = b_S - \max(0, D - D - D) \left(d_S - a_S\right)
\]

and

\[
c_F' = c_F + \max(0, D - D - D) \left(d_F - c_F\right)
\]

For the minimal workload, the area covered by the projection of the necessity distribution is:

\[
r.\left(b_F - c_S + a_F - d_S\right)/2 = r.H.
\]

The reduced minimal profile, defined by \((c_S, d_S, a_F, b_F)\), is used when \( w \leq D < D \). Values \( c_S' \) and \( b_F' \) are:

\[
c_S' = c_S + \max(0, D - D - D) \left(d_S - c_S\right)
\]

and

\[
b_F' = b_F - \max(0, D - D - D) \left(b_F - d_F\right)
\]

Figure 11 shows an example of modified extreme profiles.

### 4.3.2 Configurations with overlap

In the configurations with small or large overlap, the maximal resource profile is treated like the configuration without overlap, because the presence possibility distributions are the same. On the other hand, the minimal resource profile is treated differently in each case.

In case of large overlap, the minimal profile is null \((N(t) = 0 \ \forall t)\), so there is no need to reduce this
and area $r$. If the workload $r.w$ is lower than the minimal workload $r.H_{\text{min}}$, we use the projection of the presence probability distribution and determine $H_{\text{min}}$ so that:

$$r.w = \int_0^{+\infty} r.p(t) dt = r.H_{\text{min}} \left( \frac{d_F - a_S}{2} + \frac{c_F - b_S}{2} \right).$$

In case of a small overlap, the area of the projected necessity distribution is:

$$r.\hat{D} = r.\beta_0 \frac{b_F - c_S}{2} = r \frac{(b_F - c_S)^2}{2(d_S - a_F + b_F - c_S)}.$$

If this area is lower than the minimal workload $r.w$, we use the projection of the presence probability distribution and determine $H_{\text{min}}$ so that:

$$r.w = \int_0^{+\infty} r.p(t) dt = r.H_{\text{min}} \left( \frac{d_F - a_S}{2} + \frac{c_F - b_S}{2} \right) + r \frac{(1 - H_{\text{min}})^2(b_F - c_S)^2}{2(d_S - a_F + b_F - c_S)}.$$

If the workload $r.\hat{D}$ is smaller than the minimal profile area $r.\hat{D}$ (i.e. $w \leq D < \hat{D}$), we use the reduced minimal profile defined by $(c'_S, d'_S, a'_F, b'_F)$. Values $c'_S$ and $b'_F$ are:

$$c'_S = c_S + \max (0, \frac{2(\hat{D} - D)}{\beta_0} (\alpha - c_S)).$$

and

$$b'_F = b_F - \max (0, \frac{2(\hat{D} - D)}{\beta_0} (b_F - \alpha)).$$

In this case, $H_{\text{min}} = 0$.

### 4.4 Project resource usage chart

Resource usage profiles give an idea of resource needs according to uncertainty on task start and finish dates. The remaining parameter is the task duration, who conditions the workload.

During capacity planning phase, a project resource usage chart is drawn for each resource involved in the maintenance operations in order to identify overloaded resources. These charts are established by period. Resource usage $R(i)$ for a time period $[t_{i-1}; t_i]$ is the sum of the means of resource profiles over this period for all the tasks that use this resource.

Figure 12 shows an example of resource chart built for three tasks $T = 1, 2, 3$. Resource requirement are $r_1 = r_2 = 2$ and $r_3 = 1$. For each task $T_i$, a duration $D_T$ has been chosen, resulting in a workload profile (dashed lines). Integration of this profile over a period gives the mean resource workload $R_T(i)$ for this period.

![Figure 12: Periodic resource chart.](image)

Setting all the task durations $D_T$ to $w$ (resp. $z$) gives the lower (resp. upper) bound of the workload. These bounds may be useful in a global decision framework. Once the project resource usage charts are established, problems can be solved by different ways: using the floats of the tasks, modifying resource capacity (overtime, hiring people), etc. These decisions may affect other projects.

### 5 Conclusion

In this paper we have presented a fuzzy model for tactical helicopter maintenance planning. A method to establish a resource chart for capacity planning is
proposed. Future work will focus on experiments and the development of the method in order to solve capacity problems. The assumptions on task durations will be studied, and the approach will be included in a general decision framework to compare the various corrective actions to cope with resource problems. It will also be interesting to study the link between the tactical level and the operational level (project control). At this latter level, critical chain scheduling has been successfully used to solve the resource problems in maintenance centers (Srinivasan et al.). Finally, a comparison with real data from Heli-maintenance center will be done.


