TAKING INTO ACCOUNT OF FLEXIBILITY IN HIERARCHICAL PRODUCTION PLANNING

Errromdhani Ridha., Eddaly Mansour
FSEGS, route de l’aéroport km 4, Sfax 3018 Tunisie.
erromdhaniridha@yahoo.fr,
eddaly.mansour@gmail.com

Rebai. Abdelwaheb
FSEGS, route de l’aéroport km 4, Sfax 3018 Tunisie
abdelwaheb.rebai@fsegs.rnu.tn

ABSTRACT: In this paper we address to the planning problem in the agro alimentary domain. In such industry, several specific constraints should be taken into account for planning task such as the constraints of interdependencies between the products and variable production modes. Furthermore, we present the relationship between two large fields as the production hierarchical planning and the flexibility. Especially, we show that the flexibility planning should be made a priori and then integrated in the hierarchical planning process. Indeed, we have established a mathematical model according to different production levels. While taking into account real capacities of the shop and the interdependencies between the products, the results of our formulation are satisfactory in terms of quality of solution and time requirements. It’s shown that our model is able to reach all optimal solutions for all treated models and for all system levels.

KEYWORDS: Production planning, Hierarchical planning, Flexibility, Agroalimentary industries, Linear programming.

1 INTRODUCTION

The agro alimentary industries constitute the link between the agriculture and the consumers. These industries can be distinguished by their dynamic environment due to both short cycle life and the existence of unexpected command of raw materials and products (Erol, 1999). Furthermore, the planning task in this domain is subjected to several specificities because of the high degree of incertitude and imprecision which affect the data level and the product interdependencies (Axäster and Jönsson, 1984). Referring to Schneeweiss (2003), the flexibility can be defined by “the ability of a system to cope with unforeseen changes”. Thus, the firm which anticipates perfectly the future changes does not have need to the flexibility in its production planning. Conversely, the firm which has an infinite potential of flexibility does not have need to any anticipation (Erol and Dupont, 1999).

Our motivation is to formulate the flexibility as a tool to overcome the punctual changes. Our work consists in studying the need of flexibility and locating its place in the production hierarchical planning process in the agro alimentary industry. In other words, our aim is to find a mathematical relationship for the balance between two large fields such as in the production hierarchical planning process in the agro alimentary industry and the flexibility.

The choice of the hierarchical approach is supported by its ability in simplifying the global decision-making process. The decisions transferred to inferior level are considered as constraints to be satisfied or goals to be reached for superior level (Fontan et al., 2005). So, we propose a mathematical model which allows reducing and simplifying the real constraints in the agroalimentary environment. Thus, two kind of linear programming models were proposed. In the first one, we consider a production system consisted of several manufacturing stages with a general tree of N products in order to determine the produced quantities of each type of item. Whereas the second model consists in sharing the available capacity of an item from an inferior level into each obtained item processed from it to the immediate superior level.

This is done subject to satisfaction demand constraints with the objective to minimize production costs and inventory costs. Moreover, we propose a constructive method to solve these models.

The following of this article is organized as follows: Section 2 presents the planning task in the agro alimentary environment. The section 3 describes the hierarchical planning in the agro alimentary environment. The need of flexibility and its place in the production hierarchical planning process in the agro alimentary industry is analysed in section 4. In section 5, we present, in detail, our proposed formulation. In order to validate our model, in section 6, we present a real case based on the production in a butcher’s shop. Conclusions and future directions are presented in section 7.
2 THE PLANNING IN AGROALIMENTARY ENVIRONMENT

Planning consists in determining what and when to product and what are the resources that can be used. This task requires much information’s about the sale predictions, the resource capacities and others. In the context of agro alimentary industries, several specific constraints must be taken into account while performing the planning. Therefore, the cost of raw materials is not fixed and may depend on several parameters like the climatic changes (Tangour and Saad, 2006).

In addition, there exist some seasonal products where both its production and its demand present a seasonal peak (G. Buxey, 1993) which needs additional resources such as supplementary manpower or work time or also when the raw materials of some products are seasonal such as the tomato for the production of tomato tins (Gargouri, 2003). Also, the production mode in such environment is not fixed because one raw material can provide one or several products (Tangour and Hammadi, 2006). So, one resource can be used in producing of different products. Moreover, the semi-finished and finished products are sometimes interdependent. For instance, we can find the case where we cannot produce a finished product x without producing another product y. So, even if the demand concerns one variety of products we must produce a whole range. In this case, the firm must stock finished or semi-finished products subject to limit delay which is sometimes so short and thus these firms will be obligated to reduce their sale prices (Gargouri and Hammadi, 2003). Also, the inventory cost could be increase if the product under consideration remains a long time in the stock. This is due to a loss of price or weight of the product following the freeze operation. Finally, the agroalimentary products are often characterized by a consumption delay (CD) (Tangour and Hammadi, 2006). The firms must consider not only this deadline, but also they must take into account a selling delay (SD) for these products. Generally, \( SD = \frac{1}{3} CD \), this delay gives a short time for the firm which may be one or two days.

3 HIERARCHICAL PLANNING IN AGROALIMENTARY ENVIRONMENT WITH INTERDEPENDENCIES BETWEEN PRODUCTS

In the literature, few papers have treated the production planning problems in agroalimentary environment while taking into account the case of perishable products and or the interdependencies between the products. Tadei et al. (1995) have proposed two-steps based heuristic for solving both production planning and scheduling problems in an agroalimentary firm in Portugal. The problem is subdivided into two sub-problems the annual planning and the short run scheduling. The authors have considered the minimization of two conflicting criteria such as the inventory cost and manpower cost. Gargouri et al. (2002) have proposed a real-time approach based on preferences and priority rules leading to consider the problem constraints like the minimization of costs of perishable products and the lags between the termination date and the expedition date of products. Houba et al. (2000) presented a modelling approach which reduces the requisite effort in order to develop a decision aid system for the agro alimentary industries based on constraints satisfaction techniques of which the most of decision-maker handle it manually. Their model has been applied in salad plant. Van Wezel et al. (2006) have proposed a bottleneck of scheduling flexibility in the transformation of the food industries.

In the agro alimentary industries, and especially in the butcher’s shop, some products can be, in the same time, finished products destined directly for selling or semi-finished which can be transferred and/or decomposed into other sub-products. For instance, the thighs can be directly sold or decomposed into top of thighs and pestle. In other way, the production of a product A releases the production of the product B or its derivates even when there is no demand for the second product. For example, it’s impossible to produce the thighs without producing wings.

Generally, in the agoralimentary industries, we distinguish three kinds of products: finished products not decomposable, semi-finished products which cannot be sold directly and finished decomposable products which can be destined to selling or/and decomposed into other products.

Figure 1: Hierarchical nomenclature «products interdependencies »

Figure 1 presents the notion of interdependencies between products. Both the products \( X_1 \) and \( X_2 \) use the same raw material \( X \). So, through \( X \), we can produce only \( X_1 \), or only \( X_2 \), or both \( X_1 \) and \( X_2 \). \( X_3 \) is a non-decomposable finished product directly intended for selling whereas \( X_3 \) is a semi-finished product which cannot be sold directly. The latter may be transformed into \( X_{21} \), \( X_{22} \) and \( X_{23} \) such that we cannot produce \( X_{21} \) without producing \( X_{23} \), but it’s possible to produce \( X_{23} \) only without \( X_{22} \) and \( X_{21} \).
4 THE NEED OF FLEXIBILITY AND ITS PLACE IN HIERARCHICAL PLANNING IN AGROALIMENTARY ENVIRONMENT

The hypothesis which assumes that if a firm is perfectly flexible, it does not need anticipation is not realist. Therefore, in the real world, there is no perfect anticipation than perfect flexibility. In one hand, everything is not predictable, and in other hand several irreversibility and inertia factors may restrict the capacity of adaptation of the firm (Van Wezel et al., 2006). Moreover, effective planning requires major flexibility in the plans selection (Koontz, 1958).

In short term, the major concern of the firm is to find the best way of facing to demand evolutions or changes of market conditions. Berny (1986) has defined the flexibility of the firm as a response power, as soon as possible, to a random or unpredictable changes of the demand. The planners aim to maintain, as much as possible, the stability leaded by the planning, which is its main contribution, this is done while allowing to rapidly respond to environment changes (Mehrabi et al., 2000). In order to present the need of the flexibility in the firm, we refer to the approach presented in (Corrêa, 1994). Therefore, two main factors may cause the resort to the flexibility such as the uncertainty and the variability.

4.1 Uncertainty

Several studies have addressed to the uncertainty of the firm’s environment as a major cause of the need of the flexibility (Grabot et al., 2005, Corrêa, 1994; Gerwin, 1986; Slack, 1989; Swamidass and Newell, 1987). Cohendet and Llerena (1990) show that the flexibility notion is little relevant under certain environment whereas it’s an argument among others for the decision making in the case of uncertainty. So, it becomes a fundamental aspect for uncertain future. Referring to the works of Luce and Raiffa (1957) and Corrêa (1994), we define the risk as a situation where the occurrence of the events is associated with probabilities, whereas the uncertainty is defined as a situation which cannot be associated with a distribution probability.

4.2 Variability

The variability is the second reason for using the flexibility introduced by Corrêa (1994). This notion focuses on two points. The first one concerns the variation of the range of products and the second one concern the variation of demanded quantities over considered horizon period. These two variations may lead to important consequences on the resources of the production (equipments, raw-materials, manpower). Even, if such variation over future period could be anticipated, the firm must have potential of flexibility for managing these changes. Furthermore, more the environment exerts a variety of pressures on the firm; more the latter must be able to provide a wide array of reactions. This is certainly possible only if the manpower and the technologies remain flexible e.g. only if their competences will be diverse and they can rapidly put in contribution.

In particular, there exist a relationship between produced quantity over time unit and the diversity of products by each type of organization. Some authors have proposed a typology of production systems on a graph flexibility-productivity, which require human interventions. These systems have a degree of flexibility depending on the ability, adaptability and know-how of human operators. An empirical study was conducted by Reix (1979) on several firms. It is shown that the flexibility expresses a firm’s ability to respond to a changing environment and uncertain. It is a dynamic concept, but involves a calculation of estimates of this firm.

5 MATHEMATICAL MODEL

The models used in different levels were constructed by aggregating successively the entities that will be produced in the direction from low to high level. The decisions of a superior level are considered as a constraint in an inferior level (Cohendet and Llerena, 1990).

The subjected entities in the model are:

- Type: consists of a set of items having the same basic raw-materials and are related by interdependency constraints. The production of items of type i followed by items of type j may require operations machine settings and cleaning. We cannot find any interdependency between the products of two different types.
- Item: corresponds to a sold finished product, decomposable finished product or semi-finished product.

We note that any item belongs to only one family, and likewise, every family is part of only one type.

5.1 Linear Modelling per Type (LMT)

In the proposed linear model for production hierarchical planning, we consider a production system consisted of several manufacturing stages with a general tree of N products. The required data are: number of planning periods, the demand per type per period, the availability of production resources during supplementary hours, the unit production period per type, a set of unit costs. The used notations are presented as follows:

The decision variables:

\[ X_i^t : \] The quantity of type \( i \) to produce over period \( t \).
\[ S_i^t : \] The stock of product of type \( i \) at the end of period \( t \).
\[ R_t : \] The number of regular working hours used over the period \( t \).
\( Q_t \): The number of supplementary working hours used over the period \( t \).
\( ST_i \): The number of units of product \( i \) subcontracted over the period \( t \).

**The costs:**

\( C_p \): Aggregate production cost of type \( i \) over the period \( t \).
\( C_s \): Aggregate inventory cost of type \( i \) over the period \( t \).
\( Ch \): Cost of a supplementary working hour over the period \( t \).
\( Cn \): Cost of a regular working hour over the period \( t \).
\( C_{it} \): Outsourcing cost per unit of product \( i \) over the period \( t \).

**The parameters:**

\( D_t \): Aggregate demand of type \( i \) over the period \( t \).
\( mn \): The total of available regular working hours over the period \( t \).
\( nh \): The number of required hours for producing one unit of type \( i \).
\( C \): The capacity of production (Kg/hour) over the period \( t \).
\( Mnh \): Maximum number of regular working hours per day.
\( Mhs \): Maximum number of supplementary working hours per day.

Then, the proposed linear mathematical model, denoted LMT, is presented as follows:

\[
\begin{align*}
\text{Min}\ f(X,S,R,O,ST) = & \text{Min} \sum_{i=1}^{n} \sum_{t=1}^{T} \left( C_p X_{it} + C_s S_{it} + C_{it} ST_{it} \right) + Cn R + Ch O \\
\text{Subject to :} & \\
S_{it} + X_{it} - D_t = S_{it} & \quad i=1,2,...,N; t=1,2,...,T \quad (2) \\
S_{it} = 0 & \quad i=1,2,...,N \quad (3) \\
\sum_{i=1}^{n} nh_i X_{it} \leq R + O_t & \quad i=1,2,...,N; t=1,2,...,T \quad (4) \\
\sum_{i=1}^{n} X_{it} \leq C_i & \quad t=1,2,...T \quad (5) \\
0 \leq R \leq Mnh_i & \quad t=1,2,...,T \quad (6) \\
0 \leq O \leq Mhs & \quad t=1,2,...,T \quad (7) \\
S_{it} + D_t = S_{it+1} + X_{it} + ST_{it} & \quad i=1,2,...,N; t=1,2,...,T \quad (8) \\
X_{it} \geq 0 & \quad i=1,2,...,N; t=1,2,...,T \quad (9) \\
S_{it} \geq 0 & \quad i=1,2,...,N; t=1,2,...,T \quad (10) \\
ST_{it} \geq 0 & \quad i=1,2,...,N; t=1,2,...,T \quad (11)
\end{align*}
\]

The objective function (1) is the sum of production costs, inventory costs, cost of total regular and supplementary working hours for all products over the horizon period. The constraints (2) are balancing stocks equations. They state that the stock at the end of period \( t \) is equal to the stocks at the period \( t-1 \) plus the produced quantity at the period \( t \) minus the demand of the period \( t \). The constraints (3) ensure that the initial stock is equal to 0 at \( t=1 \). The constraints (4) ensure that the planned production cannot exceed the available capacity over regular and supplementary working hours over the period \( t \). The production capacity is restricted to an upper bound at each period \( t \) (constraints 5). The constraints (6) and (7) require that the number of working hours cannot exceed the maximum number of regular and supplementary working hours respectively. The constraints (8) show that, in the overload case, the firm can use outsourcing. The total quantity of product \( i \) produced over given period is calculated as the sum of its production for different products. The inequalities (9), (10) and (11) are the non-negativity conditions.

The proposed model aims to determine the produced quantities of each type of item. The output of this model is considered as specific constraints to the second proposed model presented in the next section.

### 5.2 Linear Modelling per item (LMI)

This modelling consists in sharing the available capacity of an item from a level \( i \) into each obtained item processed from it to a level \( i+1 \). This is done subject to satisfaction demand constraints with the objective to minimize production costs and inventory costs. The used notations are presented as follows:

\( C_p \): The production cost of product \( ik \) over the period \( t \).
\( C_s \): The inventory cost of product \( ik \) over the period \( t \).
\( q_{ia} \): Minimum starting quantity of product \( ik \) over the period \( t \).
\( X_{ia} \): The quantity of product \( ik \) to produce over the period \( t \).
\( S_{ia} \): The stocked quantity of product \( ik \) over the period \( t \).

\( X^* \): The obtained quantity of the product \( X_i \) from the MLT. \( X_i \) is a decomposable product into \( n \) items \( X_{i\alpha} \) \( (k=1,...,n) \).

The structure of the LMI can be formulated as follows:

\[
\begin{align*}
\text{Min} g(X,S) = & \text{Min} \sum_{i=1}^{n} \sum_{k=1}^{K} \left( C_{ik} X_{ik}S_{ik} + C_{ik} S_{ik} \right) \\
\text{Subject to :} & \\
S_{ik} + X_{ik} - D_{ik} = S_{ik} & \quad k=1,2,...,N; t=1,2,...,T \quad (13) \\
\sum_{k=1}^{K} X_{ik} \leq X^* & \quad k=1,2,...,N; t=1,2,...,T \quad (14) \\
X_{ik} \geq 0 & \quad k=1,2,...,N; t=1,2,...,T \quad (15) \\
S_{ik} \geq 0 & \quad k=1,2,...,N; t=1,2,...,T \quad (16)
\end{align*}
\]
The objective function (12) consists of both the production costs, the inventory costs for all products over all periods. The constraints (13) represent the state equations of the inventory level. They indicate that the stock at the end of period $t$ is equal to the stock at the period $t-1$ plus the produced quantity over the period $t$ minus the demand of the period $t$. The constraints (14) ensure that the sum of produced quantities of sub products of $X_i$ cannot exceed $X^*_i$.

The demand of each type is obtained by aggregating the demands of finished items belonging to such type. In order to determine this aggregate demand we multiply the sum of demands of all finished items by a coefficient $\lambda$. So,

$$D_{ij} = \lambda_i \sum_{y=1}^{n_j} d_{iy}$$

will be minimized. As shown in Figure 2, first, we reinitialize the value of $\lambda$ and we solve the proposed hierarchical model. If the system provides a feasible solution, then we subtract $\lambda$ and we restart the resolution. These steps will be repeated until reaching a value of $\lambda$ where the system cannot provide a feasible solution.

Figure 2: The steps of the resolution procedure

6 CASE STUDY: BUTCHER’S SHOP FIRM

In order to validate our proposed approach, we consider a real case study in a butcher’s shop firm. Its activities consist in raising, slaughtering and selling of poultry. In our modelling, several aggregate products are considered which consist of two families: chicken and turkey. The compositions of these two families are limited to items representing similarities in their ranges in order to modelling the interdependencies between the products.

The planning horizon ($T$) is set to a week subdivided into 7 periods. Starting from the demands of finished products, we will determine the aggregate demands of each semi-finished product. Starting from the lowest level of the tree of each type and rising from one level to another by aggregating the demand of lower level. The forecasts of demands of semi-finished products are obtained by aggregation. Aggregate demand of a semi-finished product may be the sum of requests for these derivatives or the weighted sum by coefficients provided by the company. Concerning the aggregate demand of a type, it cannot be equal to the sum of its derivatives demands because of the existence of the interdependency constraints between the products and other specificities. The results of the hierarchical model for different values of $\lambda$ are summarized in Table 1.

Table 1: results of hierarchical model

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>${\lambda}$</th>
<th>${\lambda}$</th>
<th>${\lambda}$</th>
<th>${\lambda}$</th>
<th>${\lambda}$</th>
<th>${\lambda}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
</tr>
<tr>
<td>1.5</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
</tr>
<tr>
<td>1.4</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
<td>0.003 0.003 0.003</td>
</tr>
</tbody>
</table>

By using the WinQSB program, it’s shown that, for $\lambda = 1.8$, all the demands of finished products intended directly for sale are satisfied and the holding inventory costs is very high. So, it’s recommended to reduce the value of $\lambda$. By reducing this value, we observe that the demands remain satisfied but stocks are still generated then it’s possible to decrease $\lambda$. We have solved the hierarchical model by setting $\lambda$ to several values $\{1.8; 1.5; 1.4; 1.3; 1.29; 1.28; 1.27\}$. It’s shown that more reducing the value of $\lambda$, more the total quantity of stocks decreases. For $\lambda = 1.27$, the model has not provided a feasible solution. So, we have returned to the last value which corresponds to a feasible solution $\lambda = 1.28$.

7 CONCLUSIONS AND PERSPECTIVES

In this paper, we proposed the resolution of hierarchical production planning problem in the agroalimentary industries. First, we have presented the interest of the relationship between the planning and the flexibility. Then we showed the need of flexibility face to the incertitude and the variability.

Finally, we have proposed a mathematical model related to different production levels. The results show that our model is able to reach optimal solutions for all system levels with minimum cost. The advantage of this model is related to its flexibility and adaptation to all new situations. Therefore, it may be used by the decision maker
for more extended horizon periods.
As future directions, several other problems can be treated in more depth as the problems related to breeding, subcontracting and distribution of finished products.

REFERENCES


