OPTIMAL PRODUCTION PLANNING FOR A MULTI PART-TYPE MANUFACTURING SYSTEM WITH PRODUCTION RATE DEPENDENT FAILURE RATES

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ABSTRACT: *This study deals with the problem of dependence between production and failure rates within the context of multi part-type production system. It gives an answer about how to produce, i.e. the production rates, and what to produce, i.e. the part-type selection, over a finite horizon of H periods of equal length.*
We consider a single randomly failing and repairable machine that produces a multi part-type. The first part-type is produced to meet a strategic demand d(k) via a buffer stock S, each period k (k=1, 2, …, H) . The second part-type is considered as a secondary demand in order to maximize the profits the production system. It is produced during an interval at the end of each period k of the horizon H. Our objective is to determine the production planning that maximizes the total profit. This optimal planning includes the optimal production rate during each period and the optimal duration of the production interval of the second part-type.

KEYWORDS: *Production control, failure rate, inventory, multi part-type.*

1 INTRODUCTION

Production systems are submitted to uncertainties and disturbances like demand rates or failures. In this context, there has been increasing interest to study maintenance policies and production control strategies under a combined approach. This approach has been studied in (Boukas et *al.*, 1990), (Gharbi and Kenne, 2000) and (Srinivasan and Lee, 1996), etc…. All these articles propose a joint optimization of maintenance management and production control.
(Buzacott and Shanthikumar, 1993) proved the importance of the maintenance policy in the minimization of the generated costs. Van der Dyun Schouten and Vanneste (1995) studied a preventive maintenance policy based on the production unit age and the stock capacity.
In (Meller and Kim, 1996) authors studied the impact of the preventive maintenance on a system of two machines and an intermediate buffer with a fixed capacity. (Sarker and Haque, 2000) proposed a simultaneous optimization of strategic stock and the maintenance policy.
Irvani and Duenyas (2002) presented an integrated maintenance/repair and production/inventory model using a Markov Decision Process. (Benbouzid et *al.*, 2003) presented a comparative study on strategies of joint scheduling of maintenance/ production in flowshop workshops. Kenne and Gharbi (2004) studied the stochastic optimization of the production control problem with corrective and preventive maintenance. Chelbi and Ait Kadi, (2004) presented a joint strategy of production and preventive maintenance for a randomly failing production unit. Kuo (2006) studied a joint maintenance and product quality control problem over a finite horizon. Panagiotidou and Tagaras (2007) developped a model for preventive maintenance optimization in the production process. (Aghezzaf et *al.*, 2007) proposed a model to determine an integrated production and maintenance plan that minimizes the expected total production and maintenance costs over a finite planning horizon.

In this study we consider the production control and failures of the production unit under a joint approach. Some studies have examined the relationship between production rates or planning and failures rates.
(Martinelli, 2005) studied a manufacturing system with a single machine producing a single part-type and a finite capacity inventory, with failure rate depending on the production rate. Author considered a machine characterized by a Markov failure/repair process with two different failure rates, one for low and one for high production rates. (Liberopoulos and Caramanis, 1994) studied the optimal flow control of single-part-type production systems with production rate dependent failure rates. In (Dahane et *al.*, 2009) we were interested to dependence between failure and production rates, under uncertainties of the stock level. We determined the mathematical model of the optimal production planning.

In this work we study the problem of production planning for a manufacturing system, producing a multi part-type, when failure rate depends on the production rate. The objective is to determine the production planning over a finite horizon maximizing the generated profits.

The rest of the paper is organized as follows. Next section presents the considered system, notation and working assumptions. In section 3 we develop the mathematical model, basing on expressions of generated costs and benefits. Section 4 is dedicated to the numerical procedure and the solving genetic algorithm. Summary and conclusion are provided in section 5.

2 NOTATION AND WORKING ASSUMPTIONS

The considered multi part-type manufacturing system consists on a randomly failing machine M, which produces a part-type P_a with a maximum production rate U_{max} to meet a demand $d(k)$ at the end of each period k of the production planning H (k=1,..H).
The part-type P_a is considered as the strategic part, intended to satisfy the direct customer. But in order to maximize the exploit of the machine's production capacity it was decides to expand the manufacturing system activities to a new segment, by producing another product P_b, more attractive and more profitable.

In this problem, the planning horizon is divided into H periods with the same length Δt (figure 1). During each period the production of part-type P_a is performed over the interval ΔP_a, with $\Delta P_a < \Delta t$.
On the other hand, part-type P_b is produced at the end of the period, during the interval ΔP_b with $\Delta P_b < \Delta P_a$, $\Delta t = \Delta P_a + \Delta P_b$. (see figure 1).

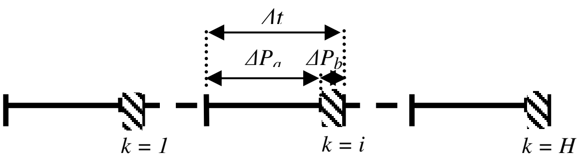

Figure 1: Production planning horizon

We consider dependence between the production and failure rates, such as the failure rate $\lambda_k(t)$ during each period k (k = 1, 2, …, H) depends on the production rate $U(k)$. So, the production at high rate accelerates the machine degradation and therefore increases the number and the total cost of repairs. But producing at low rates contributes to an increase of the probability to incur losses due to unsatisfied demands (or delays).
Our objective is to determine the optimal production rate $U(k)$ for each period (k = 1, 2, …, H) and the optimal duration of the production of the secondary part-type P_b, which maximize the total benefit over the planning horizon $H\Delta t$.
The total benefit consists on the difference between profits including production profit of each part-type and the generated cost that includes inventory, maintenance, production costs and delay penalties.

The following notation is used in the formulation:

$f(.)$ Probability density function associated to machine time to failure.

$F(.)$ Probability distribution function associated to machine time to failure, such as:

$$F(t) = \int_0^t f(x)\,dx$$

Δt Period duration

$d(k)$ Demand of part-type Pa during period k, k = 1,..H

$U(k)$ Production rate during the period k, k = 1,..H

U_{max} Maximum production rate, k = 1,..H

$S(k)$ Inventory level part-type Pa, at the end of period k, k = 1,..H

$\lambda_{max}(t)$ Nominal machine failure rate function (with $\lambda_{max}(0)$=0)

$\lambda_k(t)$ machine failure rate function during period k, k =1,..H

C_h^a Holding cost of one product of Pa during one time unit

C_h^b Holding cost of one product of P_b during one time unit

C_p^a Production cost of one unit of P_a

C_p^b Production cost of one unit of P_b

P_r^a Production profit of one unit of P_a

P_r^b Production profit of one unit of P_b

C_r Minimal repair cost

IC The total average inventory cost over the planning horizon $H\Delta t$

MC The total average maintenance cost over the planning horizon $H\Delta t$

PC The total average production cost over the planning horizon $H\Delta t$

DP The total average delay penalties over the planning horizon $H\Delta t$

PR_a The total average production profit of Pa over the planning horizon $H\Delta t$

PR_b The total average production profit of Pb over the planning horizon $H\Delta t$

PR The total average production profit over the planning horizon $H\Delta t$ $(PR=PR_a+PR_b)$

TB The total average benefit over the planning horizon $H\Delta t$ $(TB=PR-(IC+MC+PC+DP))$

To describe the problem more clearly, we formulate the following assumptions:

- Failures are detected instantaneously.
- The repairs duration is negligible.
- The produced items are imperishable with time.
- The time to switch between part-types is negligible.
- During the interval ΔP_b, machine produces at maximum rate U_{max} under a just-in-time configuration, in order to meet the maximum quantity of P_b.
- The direct demand (of part-type P_a) arrives at the end of each period. The average demand on the horizon H is

less than the maximum rate of production. But for a given period i (i = 1 .. H), we may have a demand $d(i)$ that exceeds the maximum production rate Umax:

$$\begin{cases} \overline{d(k)} \le U_{\max} \\ \exists i = 1, H : \ d(i) > U_{\max} \end{cases}$$

- If the demand is not fully satisfied, the amount recovered is necessarily satisfied during the following period:

$$\frac{d(k)}{U(k+1)} \le \Delta P_a \quad \forall k = 1, (H-1)$$

If the entire demand at the end of the last period H is not met, an additional interval ΔTA is added in order to recover the remaining quantity with the maximum production rate U_{max}:

$$\Delta TA \le \frac{d(H)}{U_{\max}(k+1)}$$

- A failure to meet demand at the end of each period generates delay penalties that depend on the time required to recover the remaining quantities.

- Production rates can meet the overall demand during the horizon H:

$$\left(\sum_{k=1}^{H} d(k)\right) \le \left(\left(H.\Delta P_a.\sum_{k=1}^{H} U(k)\right) + \Delta TA.U_{\max}(k+1)\right)$$

3 MATHEMATICAL MODEL

Our objective is to determine simultaneously the optimal production rates over the finite horizon H and the optimal duration to produce the secondary part-type P_b, which maximize the total average benefit.

The total benefit is the difference between the generated profits and the generated costs. It includes the generated profit per part-type.

The total expected cost is including the production cost, the inventory holding cost, the maintenance cost and the delay penalties.

3.1 Inventory holding cost

The total inventory holding cost IC includes the inventory cost of each part-type. For the second part-type P_b the demand is infinite. So, the machine operates in a just-in-time configuration with its maximum production rate U_{max}. Consequently, the inventory holding cost for P_b is null ($IC_b = 0$).

The inventory holding cost for P_a is given by the following expression:

$$IC = IC_a = \sum_{k=1}^{H} C_h^a.Z(k)$$

$Z(k)$ is the zone generated by the inventory level evolution during the period k (k=1,..H).

The evolution of the inventory level can be expressed as:

$$S(k) = S(k-1) + U(k)\Delta P_a - d(k)$$

So,

$$S(k) = \sum_{i=1}^{k-1}\left(U(i)\Delta P_a - d(i)\right)$$

The generated zone during a period is given as follows:

$$\begin{aligned} Z(k) = {} & S(k-1)\Delta P_a + \frac{1}{2}U(k)\Delta P_a^2 \\ & + \left(S(k-1) + U(k)\Delta P_a\right)\Delta P_b \end{aligned} \qquad \forall\ k = 1,..H$$

Thus,

$$\begin{aligned} Z(k) = {} & \Delta t \sum_{i=1}^{k-1}\left(U(i)\Delta P_a - d(i)\right) \\ & + \frac{1}{2}U(k)\left(\Delta P_a^2 + 2\Delta P_a \Delta P_b\right) \end{aligned}$$

Consequently, the inventory holding cost for P_a can be expressed as follows:

$$IC = C_h^a \begin{pmatrix} \Delta t \sum_{i=1}^{k-1}\left(U(i)\Delta P_a - d(i)\right) \\ + \frac{1}{2}U(k)\left(\Delta P_a^2 + 2\Delta P_a \Delta P_b\right) \end{pmatrix}$$

3.2 Production cost

Considering the last assumption, the production unit produces exactly the requested number of part-type P_a. For the secondary part-type P_b, the machine operates at its maximum production rate U_{max}. Hence, the total production cost is given by the flowing expression:

$$PC = PC_a + PC_b$$

where

$$PC_a = C_p^a \sum_{k=1}^{H} d(k)$$

and

$$PC_b = C_p^b H\left(U_{\max}.\Delta P_b\right)$$

Thus

$$PC = C_p^a \sum_{k=1}^{H} d(k) + C_p^b H\left(U_{\max}.\Delta P_b\right)$$

3.3 Maintenance cost

The maintenance total expected cost is given by the following expression:

$$MC = C_r.M\left(H\Delta t\right)$$

where: $\Phi(H\Delta t)$ represents the average number of failures throughout the horizon H.

$$M\left(H\Delta t\right) = \sum_{k=1}^{H} \int_0^{\Delta t} \lambda_k(t)\,dt$$

We decompose the H periods of the production planning in sub-periods in order to distinguish the production interval of each part-type P_a and P_b. Let ΔP_a and ΔP_b denote the production sub-period of each part-type (see figure 2).

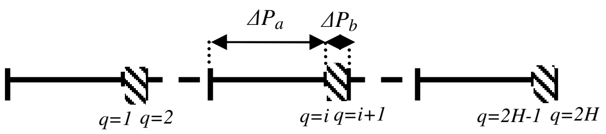

Figure 2. Production planning horizon with sub-periods

Note that odd indices q correspond to production periods P_a, whereas even indices correspond to those of the P_b.
In this case the maintenance cost and the average number of failures can be written as:

$$MC = C_r.M(2H\tau)$$

and

$$M(2H\tau) = \sum_{q=1}^{2H} \int_0^{\tau} \lambda_q(t)\,dt$$

where

$$\tau = \begin{cases} \Delta P_a & \text{If } q = 2\gamma - 1 \text{ (part-type } P_a\text{)} \\ \Delta P_b & \text{If } q = 2\gamma \quad \text{(part-type } P_b\text{)} \end{cases} \quad \Big| \gamma = 1, H$$

On another hand, the relationship between failure rates is given as follows:

$$\lambda_i(t) = \lambda_{i-1}(\tau) + \frac{U(i)}{U_{\max}} \lambda_{\max}(t) \quad \forall t \in [0,\ \tau], i \geq 1$$

From this equation we can write:

$$\lambda_i(t) = \lambda_{\max}(0) + \sum_{l=1}^{i} \frac{U(l)}{U_{\max}} \lambda_{\max}(t)$$

but $\lambda_{\max}(0) = 0$

thus

$$\lambda_i(t) = \sum_{l=1}^{i} \frac{U(l)}{U_{\max}} \lambda_{\max}(t)$$

We set apart sub-periods of each part-type:

$$\lambda_i(t) = \sum_{\substack{m=1 \\ 2m \leq i}}^{i} \frac{U(2m)}{U_{\max}} \lambda_{\max}(t) + \sum_{\substack{m=1 \\ 2m-1 \leq i}}^{i} \frac{U(2m-1)}{U_{\max}} \lambda_{\max}(t)$$

Recall that machine operates with maximum production rate U_{max} to produce P_b, so, $U(2m) = U_{max}$.
Thus, the failure rate will be expressed as follows:

$$\lambda_i(t) = \left\lfloor \frac{i}{2} \right\rfloor \lambda_{\max}(t) + \sum_{\substack{m=1 \\ 2m-1 \leq i}}^{i} \frac{U(2m-1)}{U_{\max}} \lambda_{\max}(t)$$

Now, the expression of the average number of failure throughout the horizon H is given by:

$$M(2H\tau) = \sum_{q=1}^{2H} \int_0^{\tau} \lambda_q(t)\,dt$$

$$= \sum_{q=1}^{2H} \int_0^{\tau} \left(\left(\left\lfloor \frac{i}{2} \right\rfloor + \sum_{\substack{m=1 \\ 2m-1 \leq i}}^{q} \frac{U(2m-1)}{U_{\max}} \right) \lambda_{\max}(t) \right) dt$$

So,

$$M(2H\tau) = \left(\sum_{q=1}^{2H} \left\lfloor \frac{q}{2} \right\rfloor \right) + \left(\sum_{q=1}^{2H} \sum_{\substack{m=1 \\ 2m-1 \leq q}}^{q} \frac{U(2m-1)}{U_{\max}} \right) \int_0^{\tau} \lambda_{\max}(t)\,dt$$

Finally, the maintenance total average cost can be written as follows:

$$MC = C_r \left(\left(\sum_{q=1}^{2H} \left\lfloor \frac{q}{2} \right\rfloor \right) + \frac{1}{U_{\max}} \left(\sum_{q=1}^{2H} \sum_{\substack{m=1 \\ 2m-1 \leq q}}^{q} U(2m-1) \right) \right) \alpha$$

where $\alpha = \int_0^{\tau} \lambda_{\max}(t)\,dt$

3.4 Delay penalties

Penalties are the consequence of a delay to meet all the requested demands of P_a. Figure 3 illustrates a delay situation occurred at the end of period k, which has caused a shortage recovered on the next period ($k+1$):

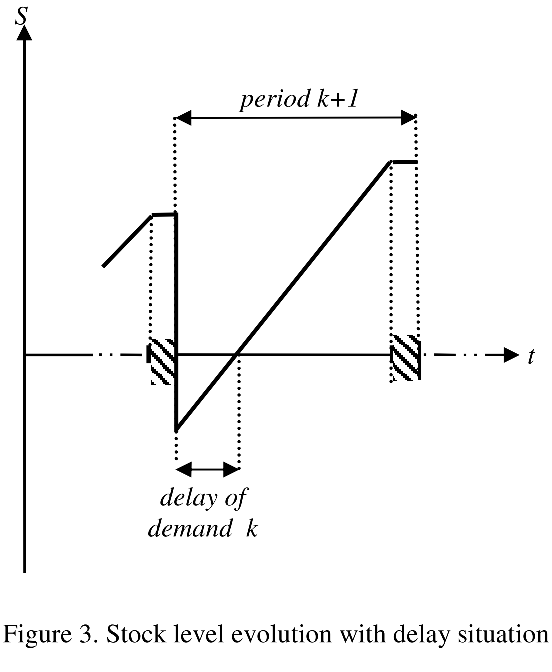

Figure 3. Stock level evolution with delay situation

These penalties are calculated according to the required duration *T(.)* to produce the missed quantity at the end of each period. They are given by the following expression:

$$CD = \sum_{k=1}^{H} C_d.\mathrm{T}(k)$$

where

$$\mathrm{T}(k) = \frac{\left|\min\left(S(k),0\right)\right|}{U(k+1)}$$

For the last period *H*, the delay is expressed as follows:

$$\mathrm{T}(H) = \frac{\left|\min\left(S(H),0\right)\right|}{U_{\max}}$$

Then,

$$DP = C_d\left(\left(\sum_{k=1}^{H-1}.\frac{\left|\min\left(S(k),0\right)\right|}{U(k+1)}\right)+\left(\frac{\left|\min\left(S(H),0\right)\right|}{U_{\max}}\right)\right)$$

The additional interval *ΔTA* (of duration *T(H)*) necessary to recovery the shortage of the last period generates inventory and maintenance overhead. Consequently, the expression of delay penalties will be given as:

$$DP = \left(\sum_{k=1}^{H} C_d.\mathrm{T}(k)\right)+\left(\begin{array}{l} C_h^a\left(\dfrac{T(H)^2}{2U_{\max}}\right) \\ +C_r\left(\begin{array}{l} M\left(2H\tau+T(H)\right) \\ -M\left(2H\tau\right)\end{array}\right)\end{array}\right)$$

where $M(2H\tau + T(H))$ denotes the average number of failures during $[0, (2H\tau + T(H))]$ (the total horizon including the additional interval).

Basing on the expression of the average number of failures obtained above (refer section 3.3), we have:

$$M\left(2H\tau+T(H)\right)=\sum_{q=1}^{2H}\int_0^{\tau}\lambda_q(t)\,dt+\int_0^{T(H)}\lambda_{\Delta TA}(t)\,dt$$

and

$$M\left(2H\tau\right)=\sum_{q=1}^{2H}\int_0^{\tau}\lambda_q(t)\,dt$$

thus

$$M=\left(M\left(2H\tau+T(H)\right)-M\left(2H\tau\right)\right)=\int_0^{T(H)}\lambda_{\Delta TA}(t)\,dt$$

where $\lambda_{\Delta TA}(t)$ denotes the failure rate during the additional interval *ΔTA*.

The relationship of $\lambda_{\Delta TA}(t)$ with the failure rate during the horizon planning is given by the following expression:

$$\lambda_{\Delta TA}(t)=\lambda_{2H}\left(\Delta P_b\right)+\frac{U_{TA}}{U_{\max}}\lambda_{\max}(t)$$

We have: $U(TA)=U_{\max}$

So,

$$\lambda_{\Delta TA}(t)=\lambda_{2H}\left(\Delta P_b\right)+\lambda_{\max}(t)$$

On anther hand, we also found that:

$$\lambda_i(t)=\left\lfloor\frac{i}{2}\right\rfloor\lambda_{\max}(t)+\sum_{\substack{m=1\\2m-1\le H}}^{i}\frac{U(2m-1)}{U_{\max}}\lambda_{\max}(t)\quad\left(\begin{array}{l}\forall t\in[0,\ \tau]\\ i\ge 1\end{array}\right)$$

Thus,

$$\lambda_{2H}\left(\Delta P_b\right)=\left\lfloor\frac{2H}{2}\right\rfloor\lambda_{\max}(t)+\sum_{\substack{m=1\\2m-1\le H}}^{2H}\frac{U(2m-1)}{U_{\max}}\lambda_{\max}(t)$$

$$=\left(H+\sum_{\substack{m=1\\2m-1\le H}}^{2H}\frac{U(2m-1)}{U_{\max}}\right)\lambda_{\max}(t)$$

Finally, the failure rate during the additional interval *ΔTA* is expressed as follows:

$$\lambda_{\Delta TA}(t)=\left(H+1+\sum_{\substack{m=1\\2m-1\le H}}^{2H}\frac{U(2m-1)}{U_{\max}}\right)\lambda_{\max}(t)$$

Consequently, the average number of failure during *ΔTA* is given by:

$$M=\int_0^{T(H)}\left(\left(H+1+\sum_{\substack{m=1\\2m-1\le H}}^{2H}\frac{U(2m-1)}{U_{\max}}\right)\lambda_{\max}(t)\right)dt$$

$$=\left(H+1+\sum_{\substack{m=1\\2m-1\le H}}^{2H}\frac{U(2m-1)}{U_{\max}}\right)\int_0^{T(H)}\lambda_{\max}(t)\,dt$$

Finally,

$$M=\left(H+1+\sum_{\substack{m=1\\2m-1\le H}}^{2H}\frac{U(2m-1)}{U_{\max}}\right)\delta$$

where $\delta=\int_0^{T(H)}\lambda_{\max}(t)\,dt$

The expression of delay penalties can be written as follows:

$$DP=\left(\sum_{k=1}^{H}C_d.\mathrm{T}(k)\right)+\left(\begin{array}{l}C_h^a\left(\dfrac{T(H)^2}{2U_{\max}}\right)\\ +C_r\left(H+1+\displaystyle\sum_{\substack{m=1\\2m-1\le H}}^{2H}\frac{U(2m-1)}{U_{\max}}\right)\delta\end{array}\right)$$

with $\delta=\int_0^{T(H)}\lambda_{\max}(t)\,dt$

3.5 Production profits

Each part produced by the machine generates a unitary profit depending on the chosen part-type (P_a or P_b).
The total profit is given by the following expression:

$$PR = PR_a + PR_b$$

where

$$PR_a = P_r^a \sum_{k=1}^{H} d(k)$$

and

$$PR_b = P_r^b.H.(U_{\max}.\Delta P_b)$$

Consequently,

$$PR = P_r^a \sum_{k=1}^{H} d(k) + P_r^b.H.(U_{\max}.\Delta P_b)$$

3.6 Total Benefit

As mentioned above, the total benefit of our manufacturing system is including the generated costs (inventory, production, maintenance and delay penalties) and production profits made during the horizon *H*. It is given by the following expression:

$$TB = PR - (IC + PC + MC + CD)$$

Basing on the expression of each term we obtain:

$$TB = \left(P_r^a \sum_{k=1}^{H} d(k) + P_r^b.H.(U_{\max}.\Delta P_b) \right) - \left(\begin{array}{l} C_h^a \left(\begin{array}{l} \Delta t \sum_{i=1}^{k-1} \left(U(i)\Delta P_a - d(i) \right) \\ + \frac{1}{2} U(k) \left(\Delta P_a^2 + 2\Delta P_a \Delta P_b \right) \end{array} \right) \\ + C_p^a \sum_{k=1}^{H} d(k) + C_p^b H (U_{\max}.\Delta P_b) \\ + C_r \left(\left(\sum_{q=1}^{2H} \left\lfloor \frac{q}{2} \right\rfloor \right) + \frac{1}{U_{\max}} \left(\sum_{q=1}^{2H} \sum_{\substack{m=1 \\ 2m-1 \le q}}^{q} U(2m-1) \right) \right) \alpha \\ + \left(\sum_{k=1}^{H} C_d.\mathrm{T}(k) \right) \\ + \left(C_h^a \left(\frac{T(H)^2}{2U_{\max}} \right) + C_r \left(H + 1 + \sum_{\substack{m=1 \\ 2m-1 \le H}}^{2H} \frac{U(2m-1)}{U_{\max}} \right) \right) \delta \end{array} \right)$$

with $\alpha = \int_0^{\tau} \lambda_{\max}(t)\,dt$ and $\delta = \int_0^{T(H)} \lambda_{\max}(t)\,dt$

4 NUMERICAL PROCEDURE

Our objective is to determine the optimal production planning, based on the optimal production rate *U(k)* of each period *k* (*k*=1,..*H*) and the optimal production interval ΔP_b of the secondary part-type, which maximise the total profit of the manufacturing system.
The adopted numerical procedure is based on genetic algorithms. Figure 4 illustrates general steps of our approach.

The "reproduction" step is based on genetic operators to maintain the genetic diversity. Each new population is generated with the standard genetic operators: Multi-point crossover with selection of the best generated chromosomes (65%) and random mutation (5%). Moreover, for each generation, the elite group is maintained (10%) and new random chromosomes are introduced (20%).

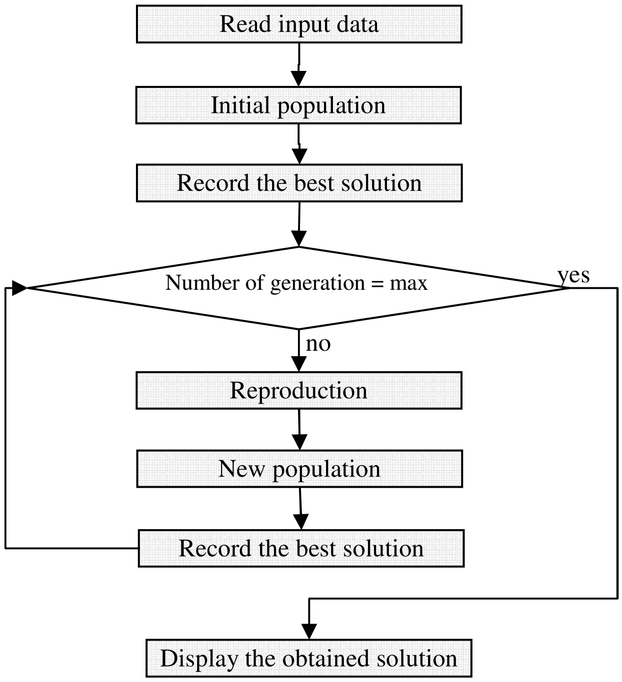

Figure 4. Approach diagram

The following figure illustrates the chromosome structure used in our procedure:

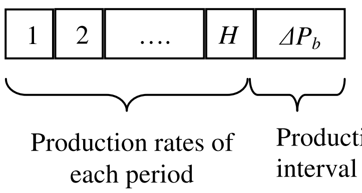

Figure 5. Chromosome structure

Each gene G_i of the first part of chromosome represents a production rate U_i, which is assigned to a period *i* of the production planning (*i* = 1, 2, …*H*). The second part of the chromosome contains the gene G_{i+H}, which represents the production interval duration of the part-type P_b

The following input data were used to illustrate our approach to find the best strategy:

- Costs: C_h^a = 0.1\$/unit/month, C_p^a = 2 \$/unit, C_p^b = 4 \$/unit, C_r = 50 \$, P_r^a = 12 \$/unit, P_r^b = 20 \$/unit.
- Demand: d = 10 units/month.
- Maximum production rate: U_{max} = 40 units/month.
- Production unit time to failure distribution $F(.)$ when machine produces at the maximum rate U_{max}: *Weibull* distribution with shape parameter 2 and scale parameter 100. In this case, we have an increasing failure rate $\lambda_{max}(t)$ = (2/100)(t/100).
- Production planning period duration Δt = 1 month.
- Number of periods (months) H = 12.

Using this procedure we obtain following results:

Period k	*d(k)*	*U(k)**
1	10	19
2	20	17
3	25	28
4	30	34
5	35	40
6	40	40
7	45	40
8	50	40
9	20	18
10	10	12
11	10	6
12	10	6

Table 1. Optimal production rates

The optimal duration of production interval of P_b is ΔP_b^* = 0.3 month.

5 SUMMARY AND CONCLUSION

This paper presents a study of a manufacturing system over a finite horizon of H periods with a single repairable machine producing multi-part-type, with dependence between the failure rate and the production rate. The considered unit aims to meet a demand at the end of each period. Our objective is to determine the optimal production planning, which includes the production rate during each period and the production duration of each part-type. A mathematical model and a genetic optimization algorithm have been developed in order to determine the optimal planning that maximizes the total benefit, basing on generated costs (inventory, production and repairs) and generated profits by each part-type.

Improvements of this work are currently under consideration, including the sensitivity of the total benefit to each part-type profit.

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